Design of Experiments for Reverse Engineering Formulations

Predicting Chemical Composition from Fewer Test Blends by Modeling Spectra using Functional Data Analysis

Tom Donnelly, PhD, CAP Principal Systems Engineer JMP Defense & Aerospace Team tom.donnelly@jmp.com





Outline

- Why use? & What is Design of Experiments (DOE)?
- What is Functional Data Analysis (FDA)?
- Reverse Engineering Case #1 Mixture of alcohol blends
- Reverse Engineering Case #2 Mineral formulations



Let's Review the Poll





Review of DOE – Why use it?

There's no better way to get the most information from the least amount of testing

- Identify important factors when faced with many
- Do sensitivity and trade-space analysis
- Optimize across multiple responses
- Characterize the operating region

d with many sis

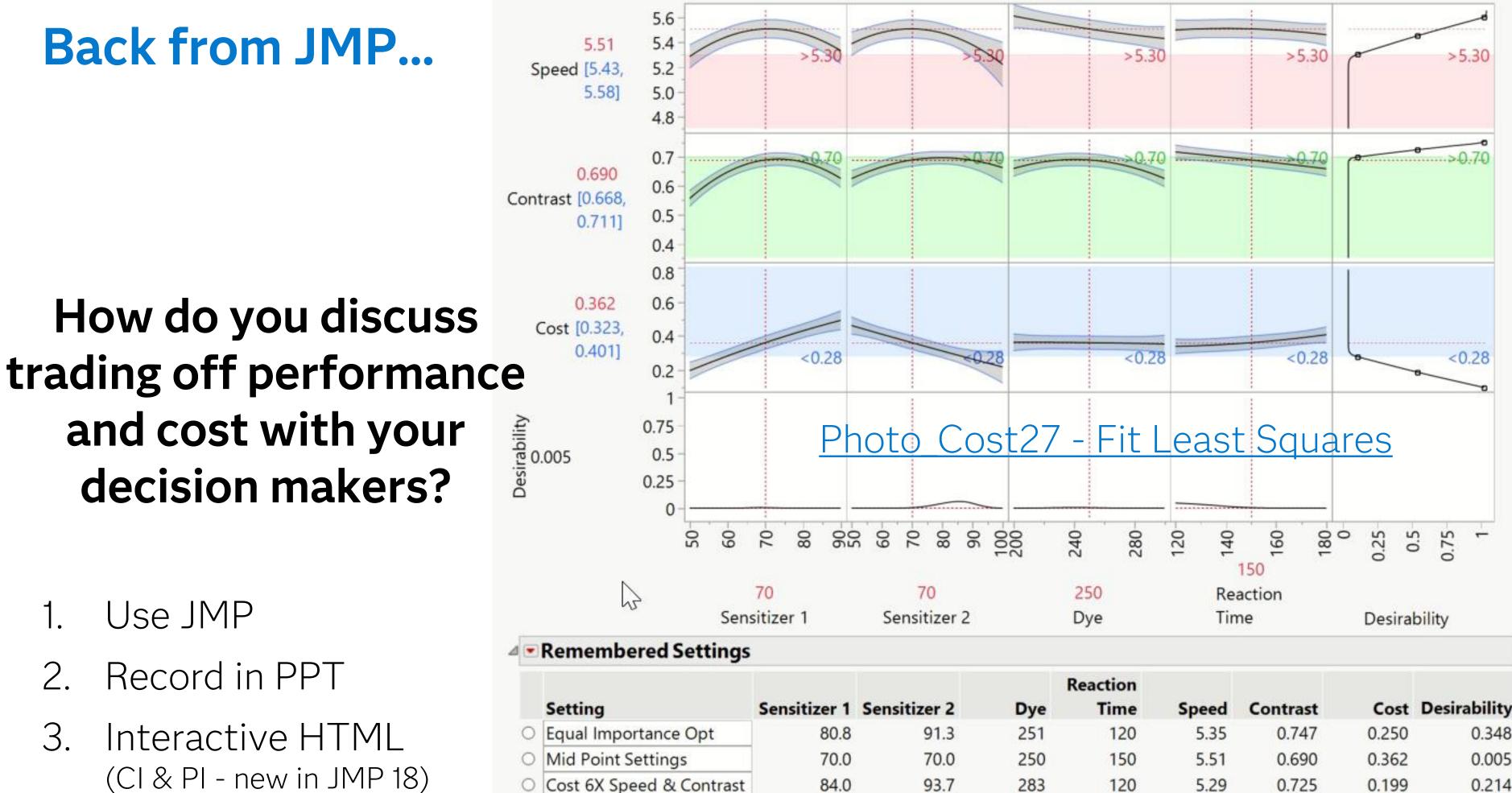




Use Prediction Profiler to Answer Questions about the DOE-Characterized/Modeled Process

Copyright © JMP Statistical Discovery LLC. All rights reserved.





93.7 84.0 82.0 90.7

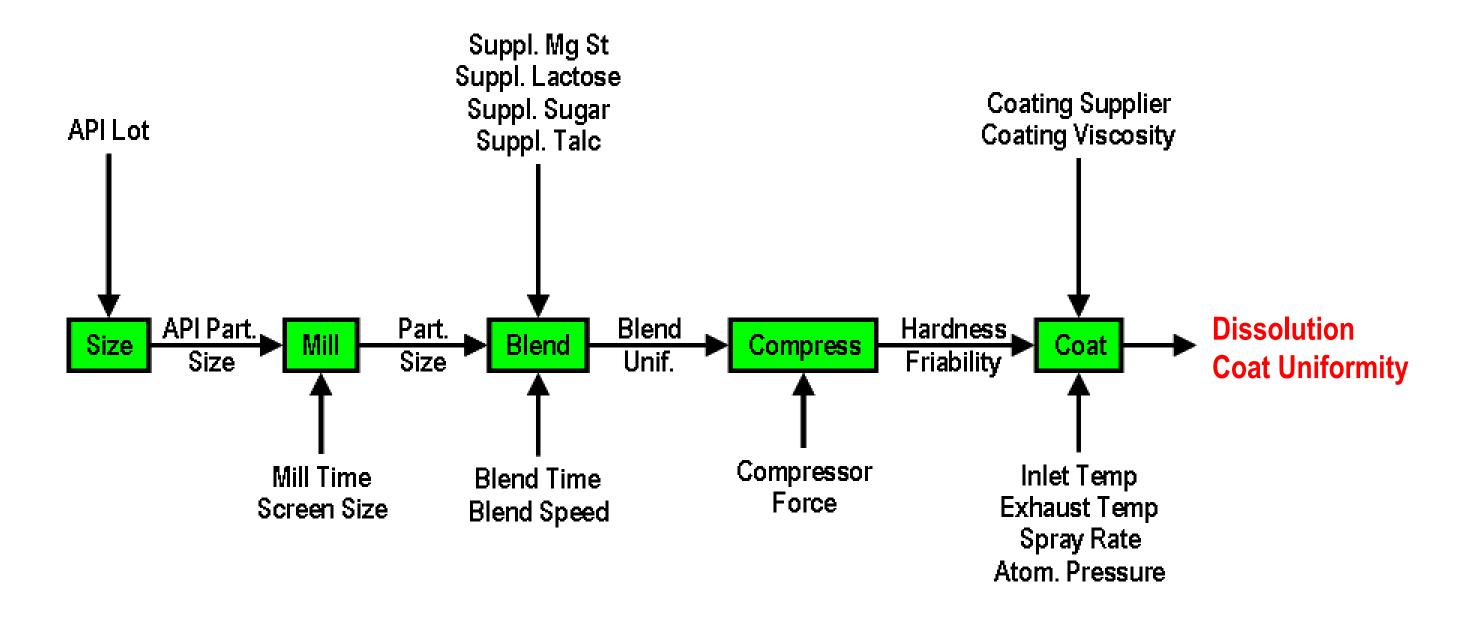
Prediction Profiler

Opt Spd3X-Cntr1X-Cost6X

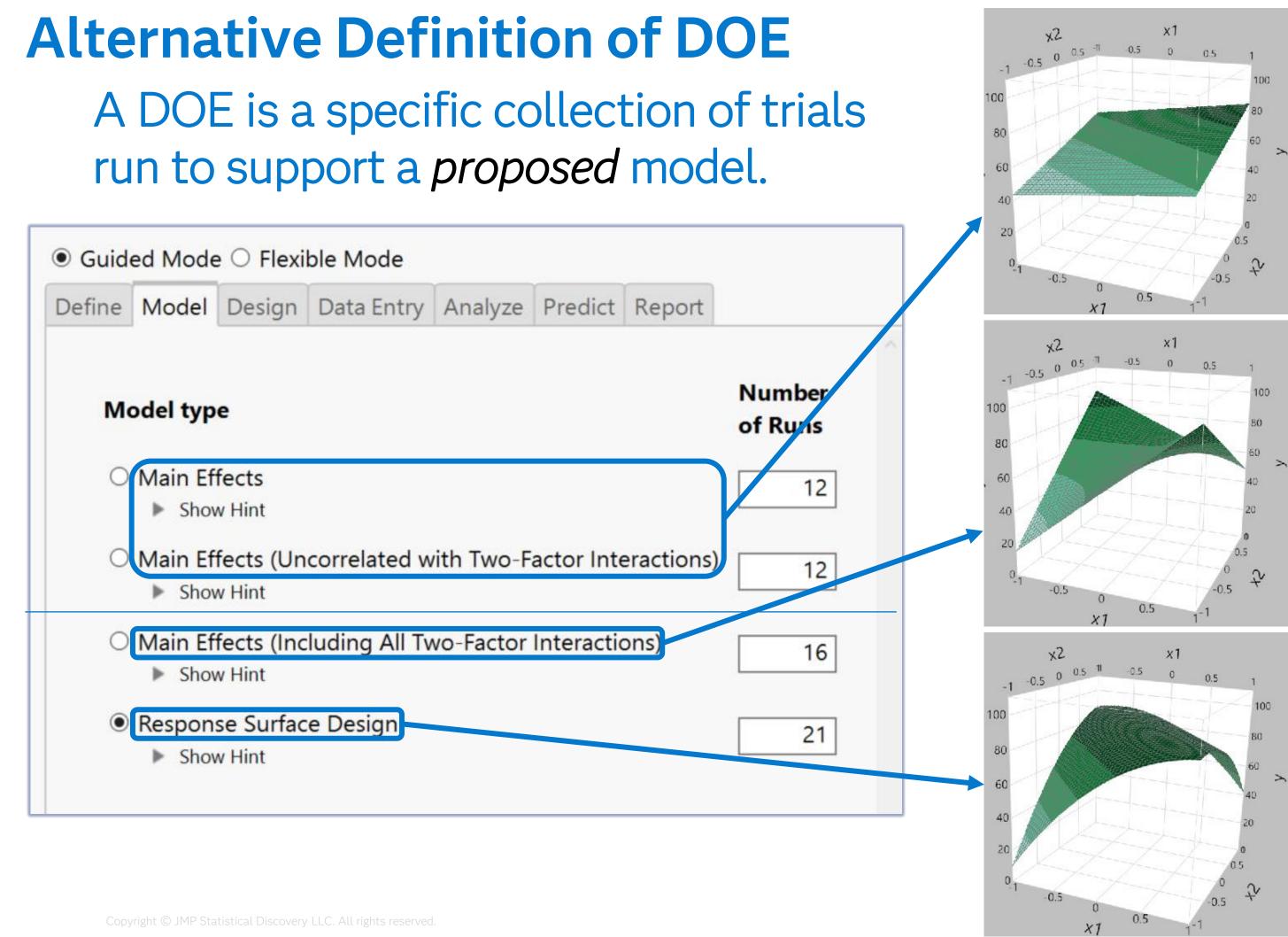
2	Dye	Reaction Time	Speed	Contrast	Cost	Desirability
3	251	120	5.35	0.747	0.250	0.348
0	250	150	5.51	0.690	0.362	0.005
7	283	120	5.29	0.725	0.199	0.214
7	287	120	5.33	0.718	0.221	0.264

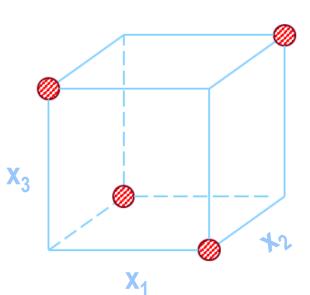
Review of DOE - Classic Definition

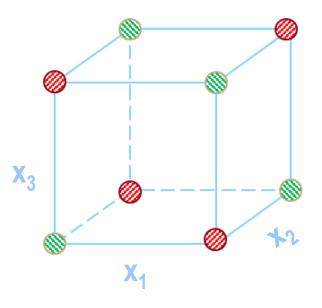
Purposeful control of the inputs (factors) in such a way as to deduce their relationships (if any) with the outputs (responses).

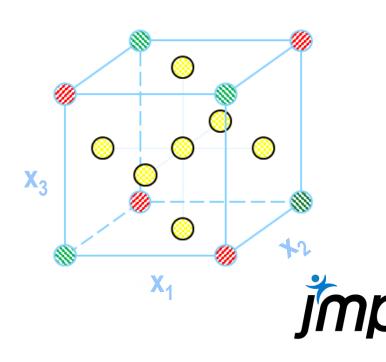












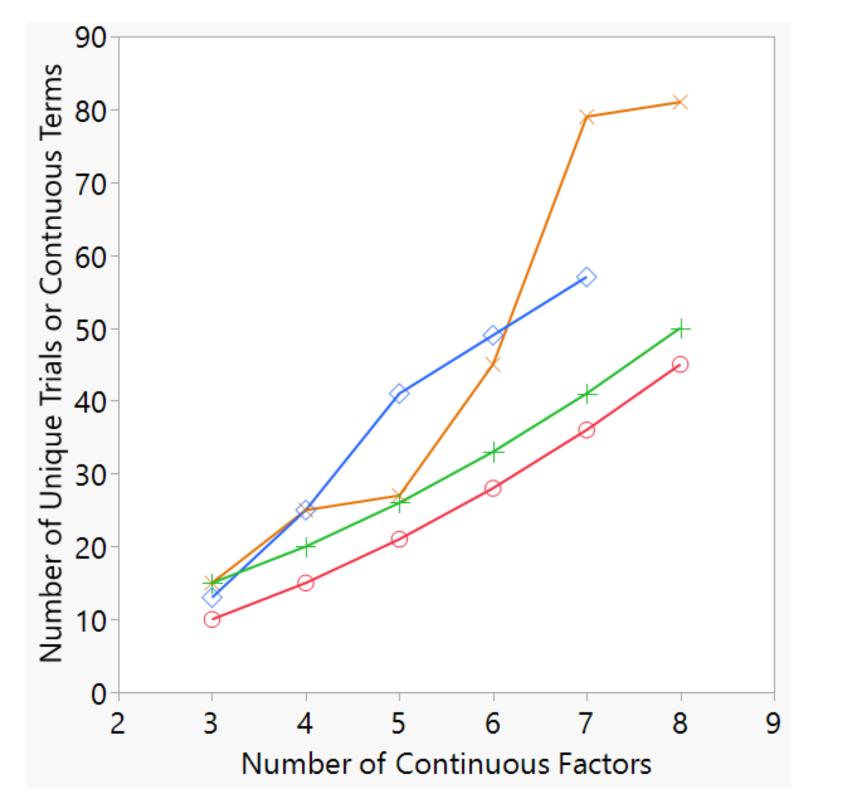
Real-World Design Issues

Model-Driven Custom DOE Makes Design Fit your Problem – NOT your Problem Fit the Design!

- Work with different kinds of control variables/factors:
 - Continuous/quantitative
 - Categorical/qualitative
 - Mixture/formulation
 - Blocking
- Work with **combinations of these four kinds** of variables
- Certain factors are hard-to-change
- Certain factor combinations cannot be run
- Want to add onto existing trials
- Need to repair broken design



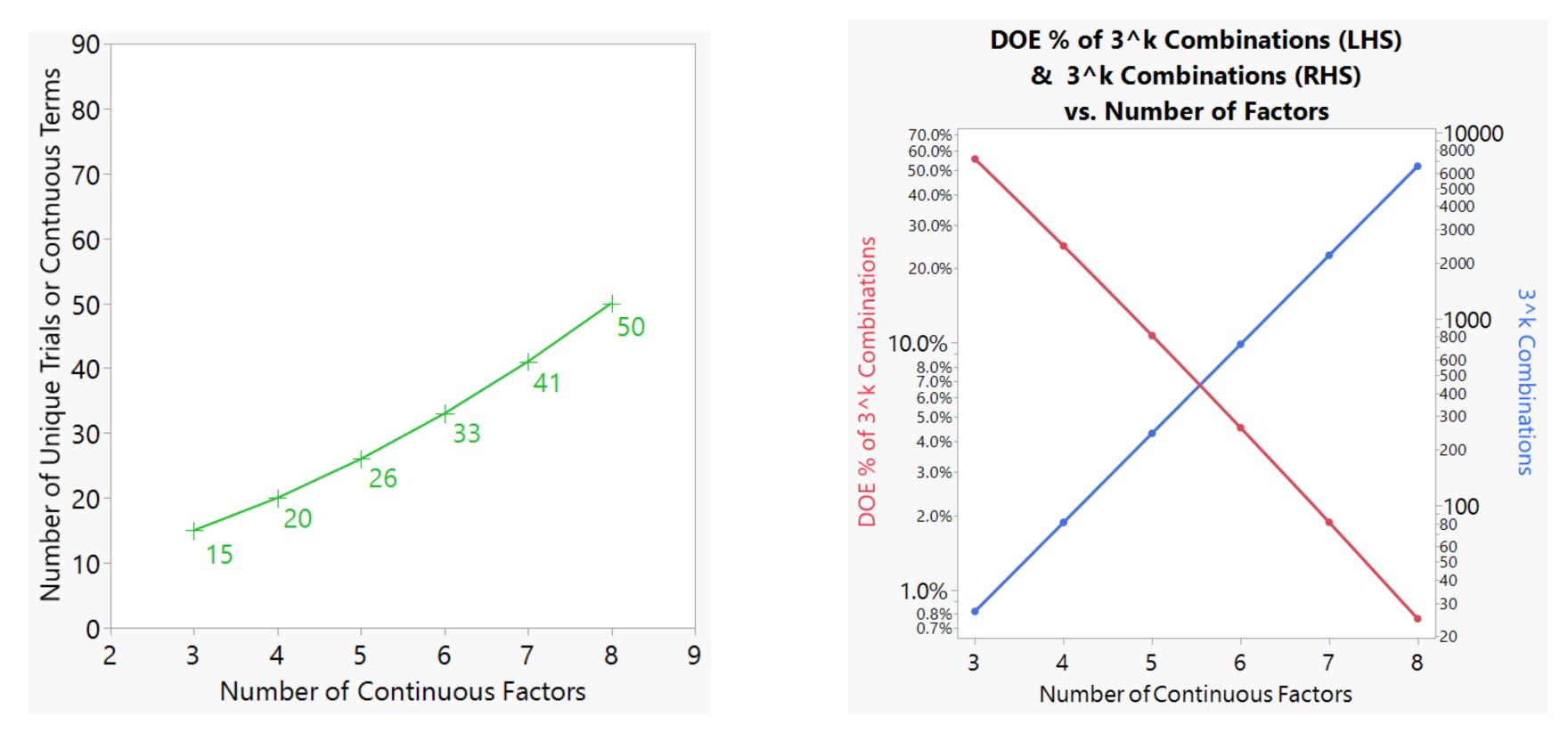
Number Unique Trials & Number Quadratic Model Terms vs. Number Continuous Factors



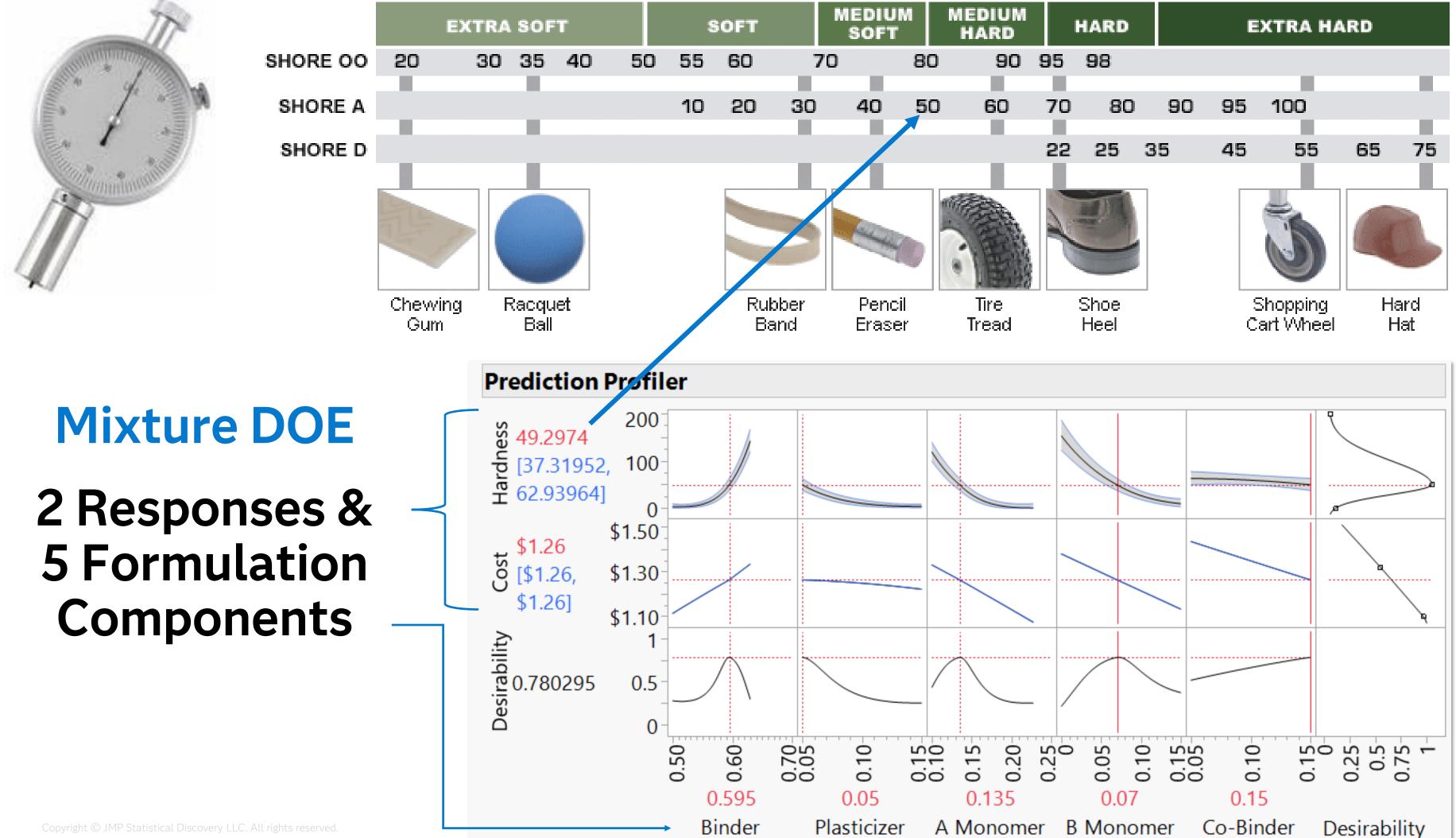
^{×—}Unique Trials in Central Composite Design
^{◇—}Unique Trials in Box-Behnken Design
^{+—}Unique Trials in Custom Design with 6 df for Model Error
[○]—Terms in Quadratic Model



Number of Custom DOE Trials Rises Slowly Number of Possible Trial Combinations Rises Rapidly (3^k) DOE Trials as a Percentage of All Combinations Falls Rapidly

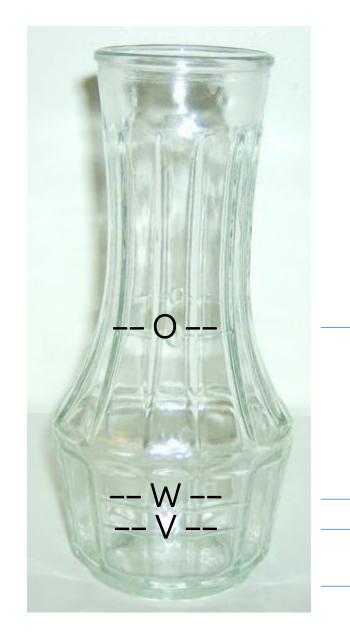






Mixture or Formulation DOE Making salad dressing is a simple 3-component blend of Oil, Water, & Vinegar





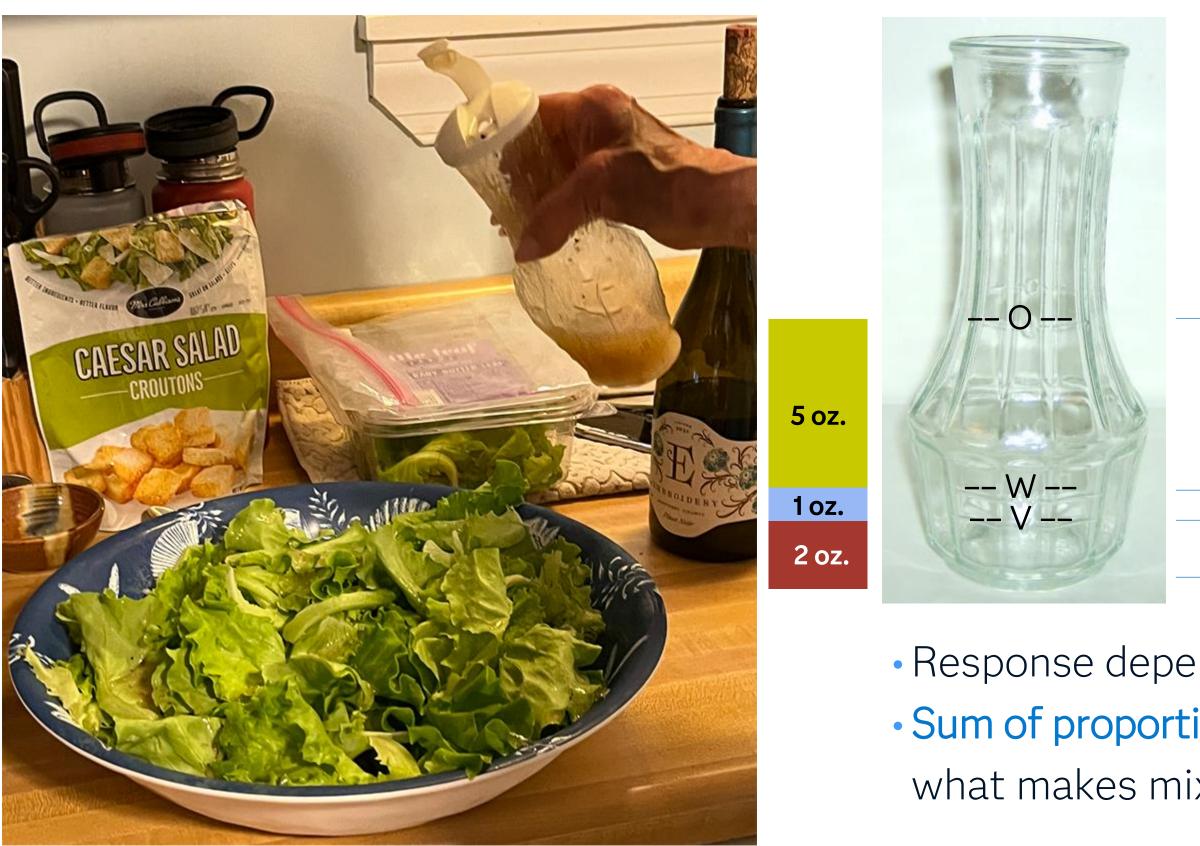
0%



• Response depends on **proportions** not quantity. • Sum of proportions equals 1. This constraint is what makes mixture DOE different.

Mixture or Formulation DOE

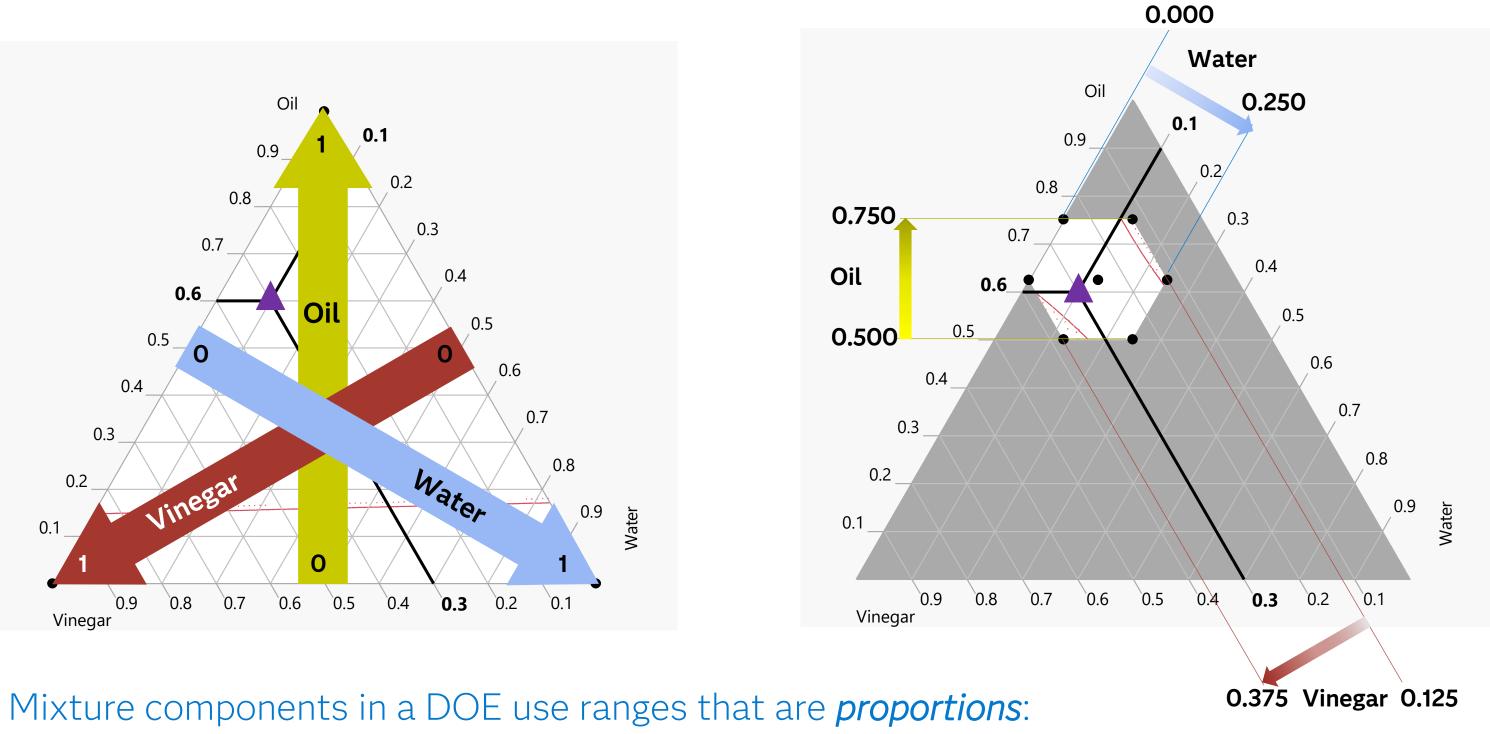
Making salad dressing is a simple 3-component blend of Oil, Water, & Vinegar





• Response depends on **proportions** not quantity. • Sum of proportions equals 1. This constraint is what makes mixture DOE different.

Use Ternary Plots to Visualize the Mixture Constraint O + W + V = 1



0.500 to 0.750 *O*: If Oil = 0.6 and Vinegar = 0.3, then 0.000 to 0.250 \mathcal{W} : Water = 1 - (0.6 + 0.3) = 0.1 (See \blacktriangle) V:0.125 to 0.375



Want to something more complex than salad dressing?



Design of Experiments for Complex Biochemical Systems

Cell-free expression (CFE) systems are a suite of methods that reconstitute complex cellular functions like transcription, translation, and metabolism outside the confines of a living cell. CFE systems have numerous biotechnological uses in sensing, biomanufacturing, medicine, basic research, and education. Most CFE systems are made from combining cellular lysates with a complex blend of excipients that improve activity. While the number of excipients makes exploring the combinatorial spaces challenging, high-throughput experimentation with acoustic liquid handling makes it feasible to optimize formulations if paired with an appropriate statistical framework.

Here we describe our use of design of experiments (DOE) to optimize excipient combinations for specific use cases of CFE. We pair our DOE with functional data analysis (FDA) to collapse activity over time measurements to metrics readily used for analysis. Initial formulation DOE examples range from five to 14 components. We further describe our efforts to push to higher scales, attempting mixture-process DOE designs with as many as 42 components using an experimental set-up that allows for 1,536 formulations to be tested at once.



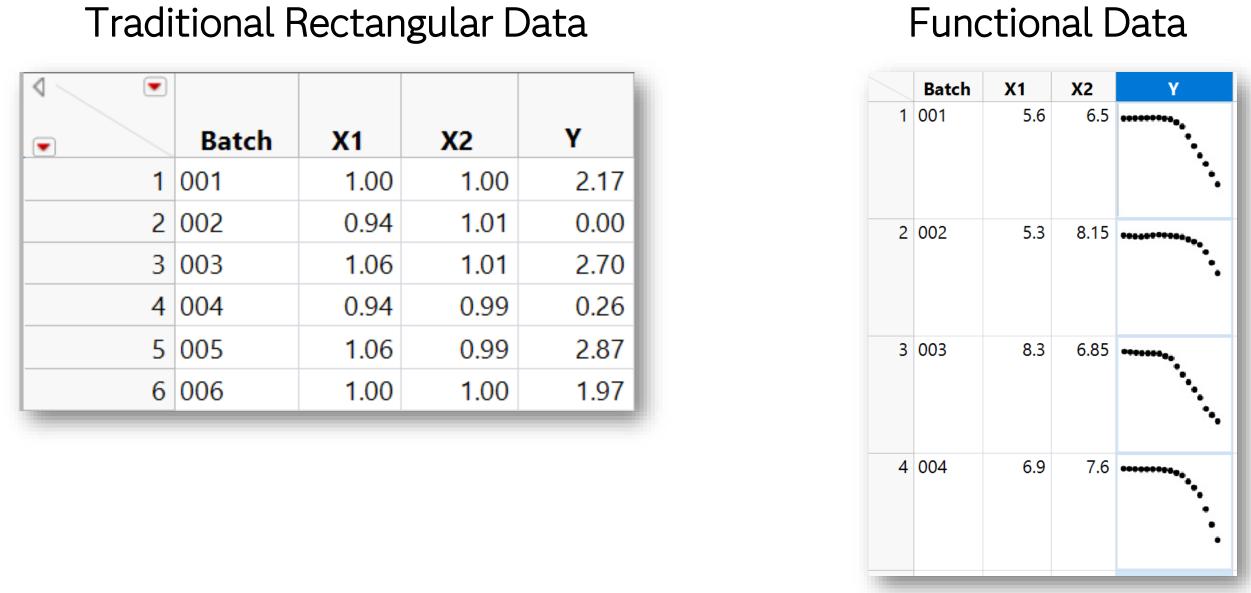


Review of Functional Data Analysis (FDA)

Copyright © JMP Statistical Discovery LLC. All rights reserved.



Functional Data Analysis (FDA) is a branch of statistics that analyzes data providing information about curves, surfaces, or anything else varying over a continuum.



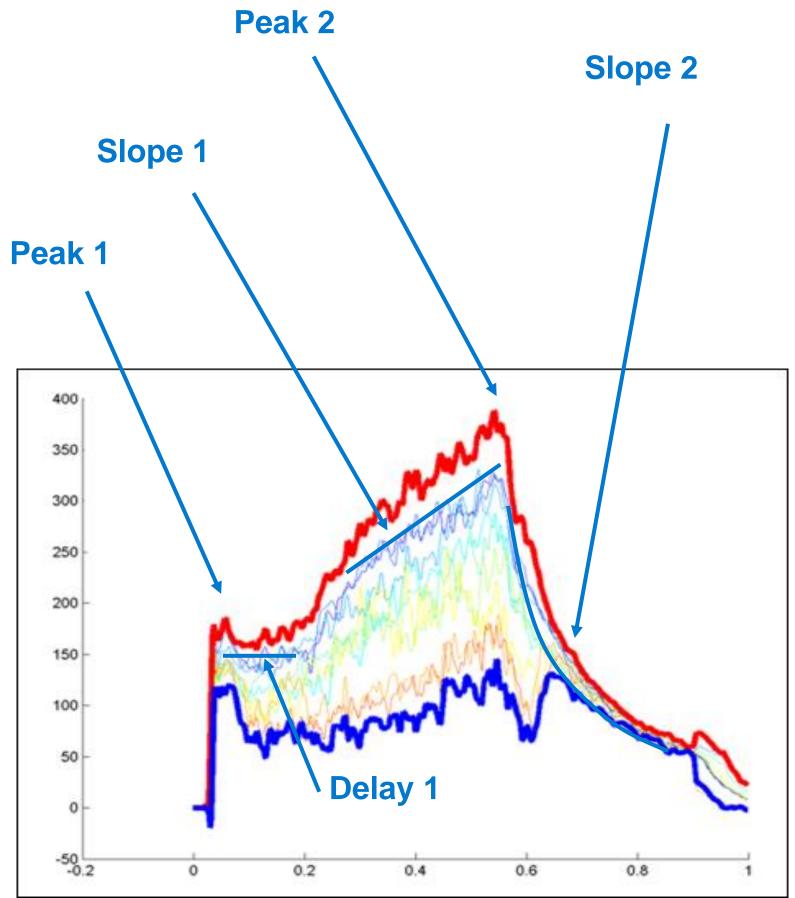
The *curve* is the fundamental unit of observation



Landmark Analysis

Moderately effective non-FDA option, but NOT as good as using FDA

Does not use information from the whole curve



Based on slide from David Harrison of Lockheed Martin Corporation

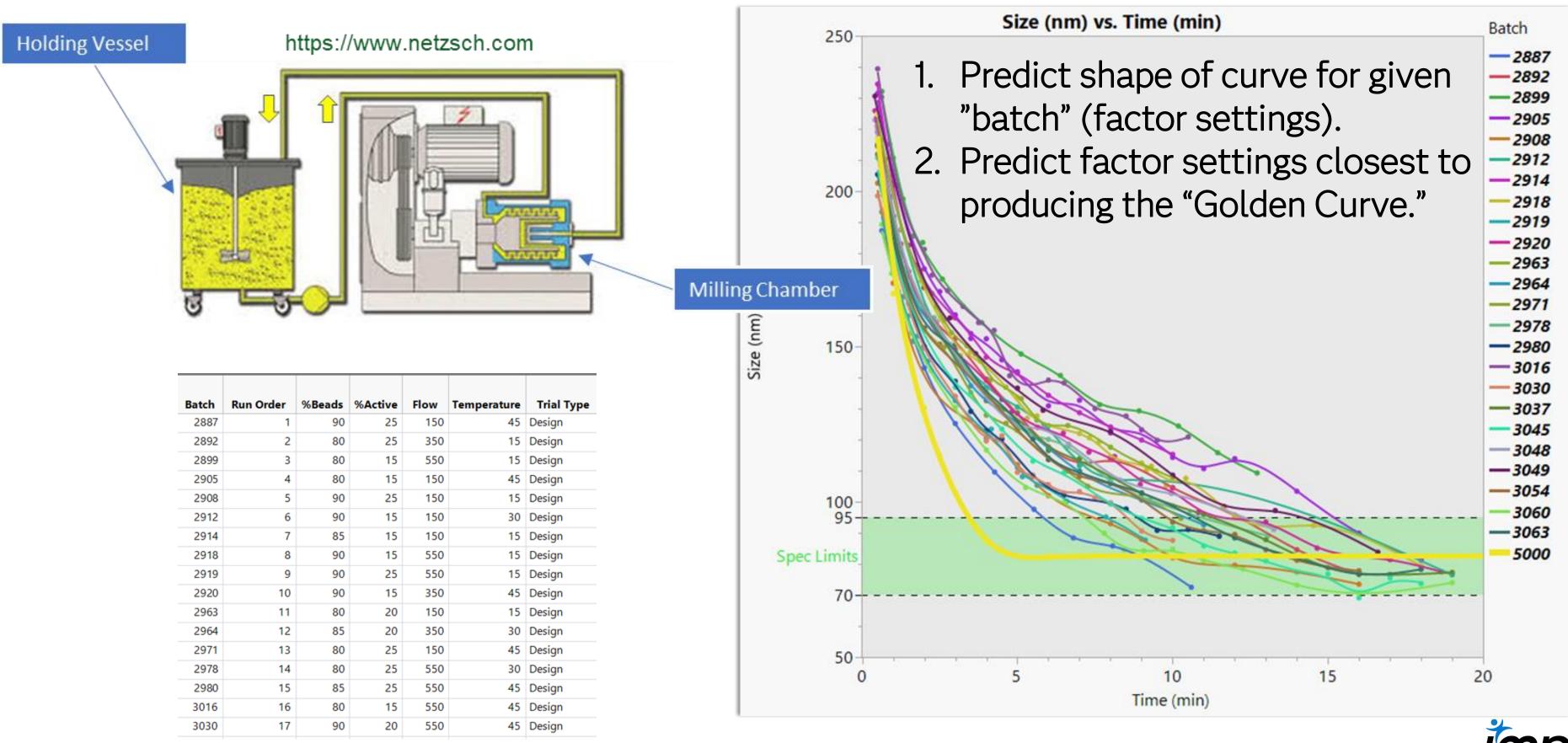


FDA Examples

- Sensor streams
- Measurements taken over a range
- Vibration signals
- Spectral data
- Radar/sonar signatures
- Trajectories of flights between cities
- Electrocardiograms (EKGs)
 - …and more!
- Almost any response in a longitudinal order



FDA-DOE Example



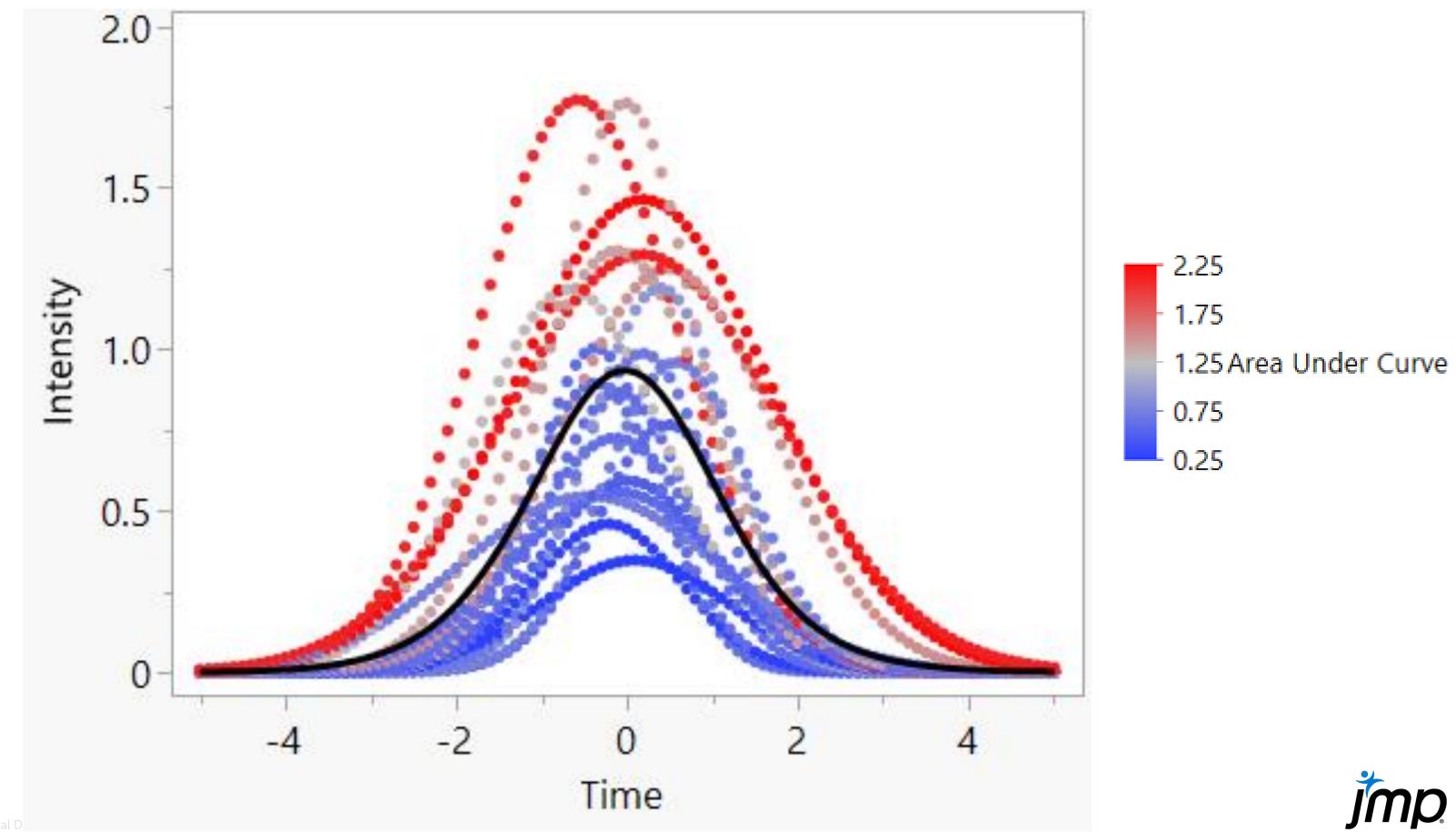


Functional Data Analysis – DOE (5-min)

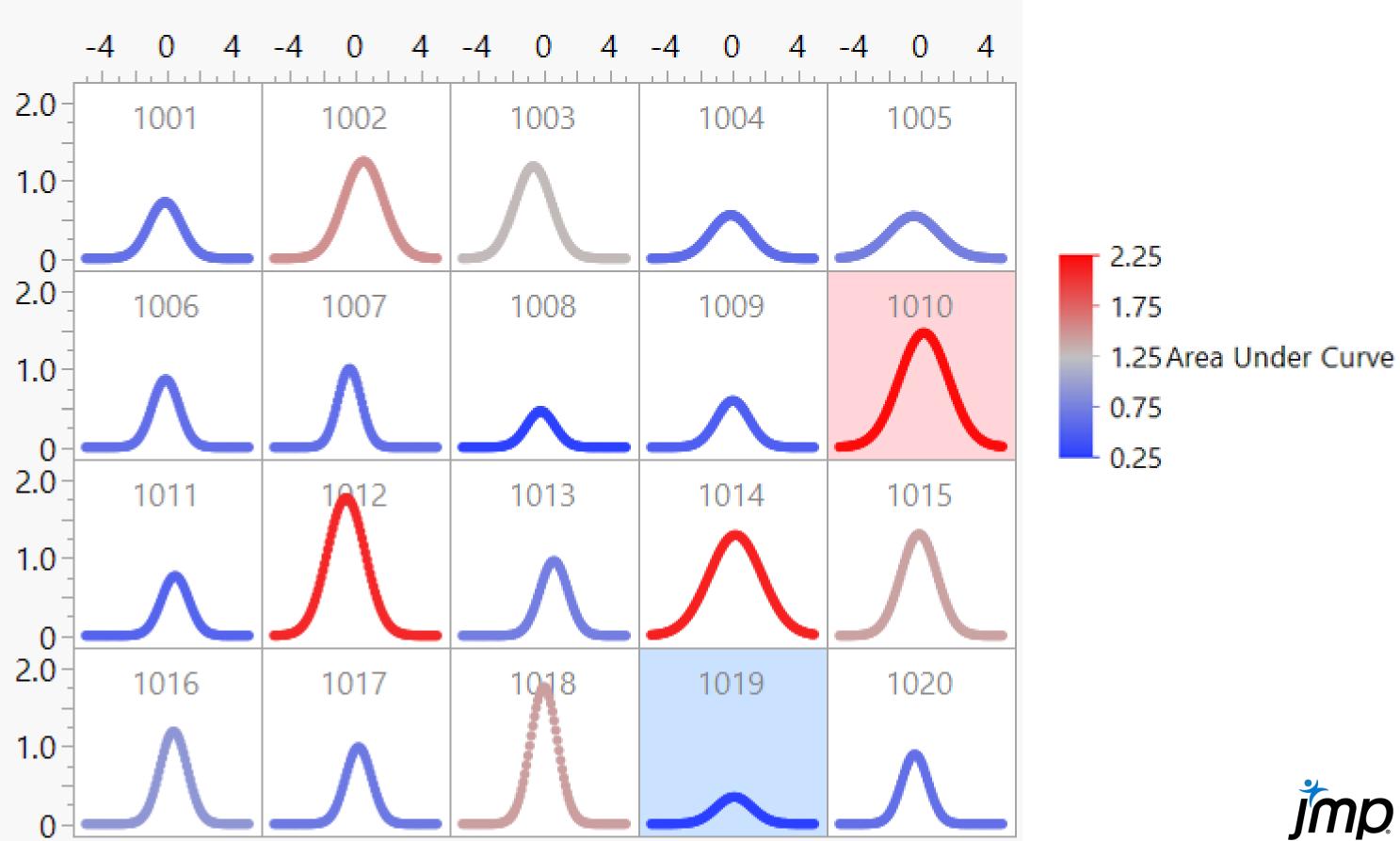
- Use Functional Principal Components Analysis (FPCA) to do dimension reduction and break "curve" data into FPC Scores & Shape Components
 - FPC Scores explain *function-to-function variation* 0
 - Shape Components explain the *longitudinal variation* 0 (e.g., *time*, *distance*, *frequency*, or *wavelength*)
- Fit models with FPC scores, cluster and graph them just like any other continuous data
- Model the FPC scores as functions of the DOE factors





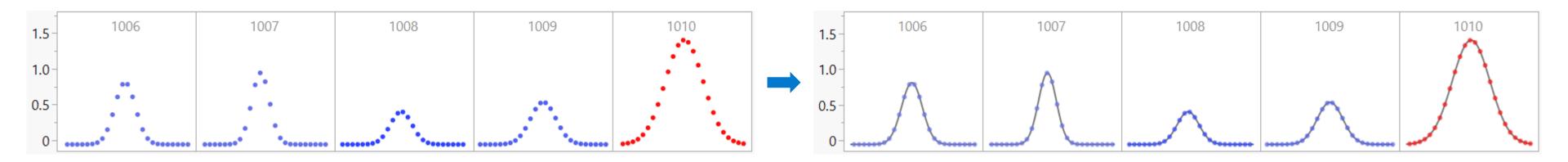




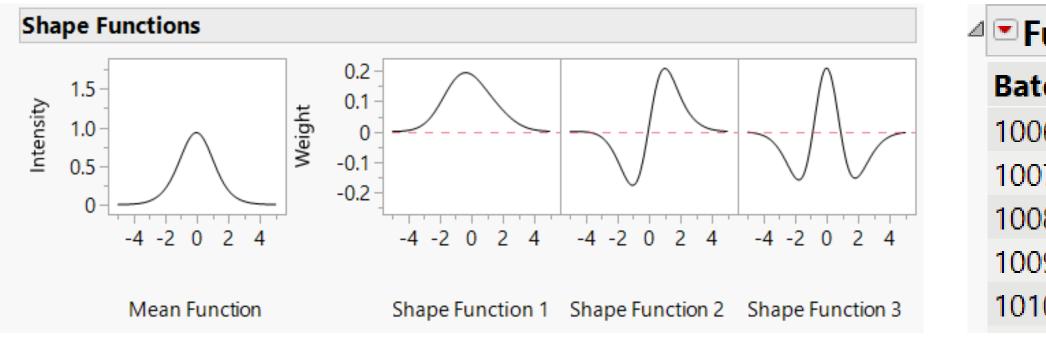




Convert streams of data into functions - "smoothly connect the dots." 1.



2. Do FPCA to create a) Shape Functions to explain the longitudinal variation. **b)** FPC scores to explain function-to-function variation.

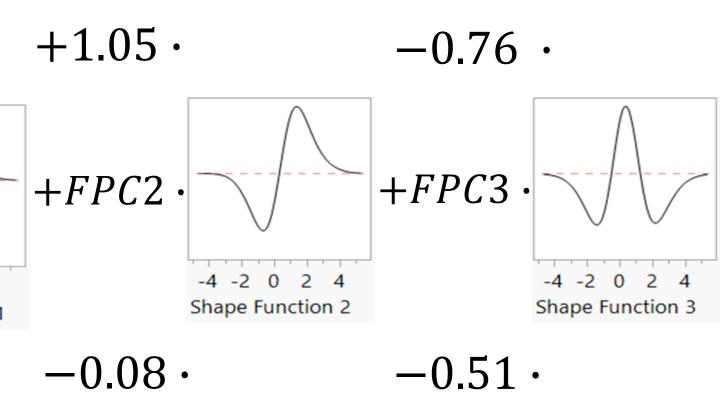


unction Summaries						
tch	FPC 1	FPC 2	FPC 3			
)6	-0.97	-0.30	0.48			
)7	-0.86	-1.13	0.54			
)8	-2.33	-0.48	-0.22			
)9	-1.61	-0.07	-0.15			
0	3.42	1.05	-0.76			

Batch	FPC 1	FPC 2	FPC 3	3. Pro	oducts of FP	C score	s multip
1001	-1.08	-0.32	0.01		ape functions		•
1002	1.39	1.77	-0.29		1	•	
1003	1.02	-1.86	-0.44	rep	produce the c	original	(batch) ı
1004	-1.42	-0.24	-0.48		Г	-	
1005	-1.20	-0.56	-0.91	1010			
1006	-0.97	-0.30	0.48				
1007	-0.86	-1.13	0.54				
1008	-2.33	-0.48	-0.22			+3.42	
1009	-1.61	-0.07	-0.15			J. T	
1010	3.42	1.05	-0.76		2.0		\bigcirc
1011	-1.53	0.99	-0.00		1.5 -		
1012	3.78	-2.15	-0.10	=	1.5 - 1.0	+FPC1	
1013	-0.91	1.57	-0.01		0		
1014	2.80	0.88	-1.07		-4 -2 0 2 4		-4 -2 0 2 4
1015	1.63	-0.21	0.40		Mean Function		Shape Function 1
1016	-0.07	1.45	0.64	1019		-2.53	
1017	-0.75	0.55	0.67			-2.33	
1018	2.14	0.15	1.90				
1019	-2.53	-0.08	-0.51				
1020	-0.91	-1.03	0.29				

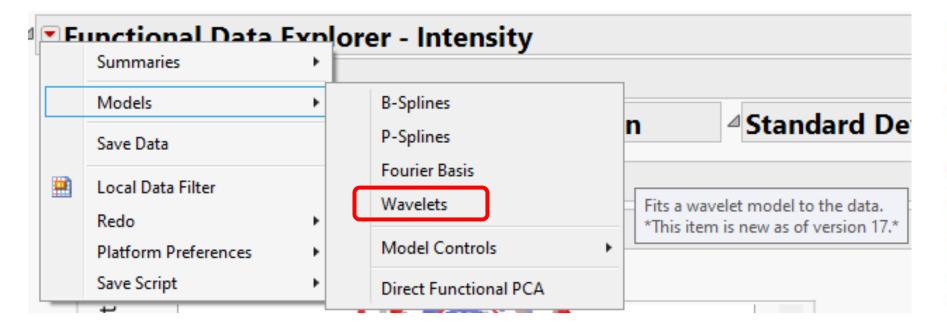


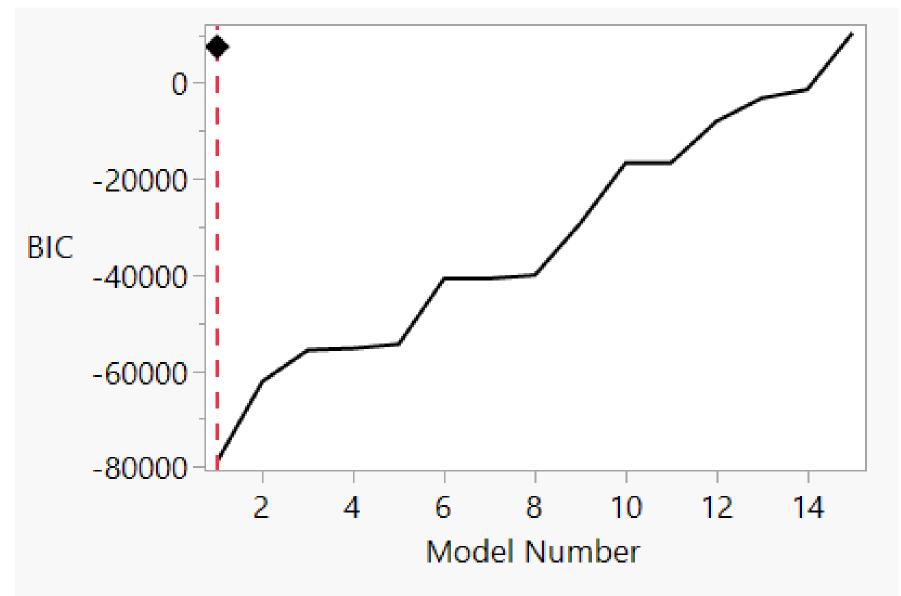
olying their corresponding to the Mean shape closely raw function curves.





Wavelets often Outperform Splines & Fourier Basis Functions



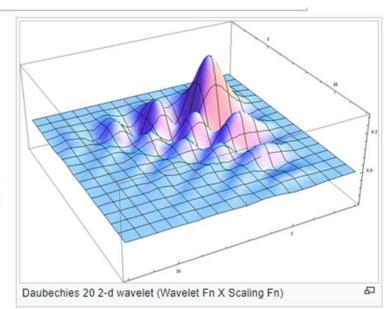


The Daubechies wavelets, based on the work of Ingrid Daubechies, are a family of orthogonal wavelets defining a discrete wavelet transform and characterized by a maximal number of vanishing moments for some given support. With each wavelet type of this class, there is a scaling function (called the father wavelet) which generates an orthogonal multiresolution analysis.

Properties [edit]

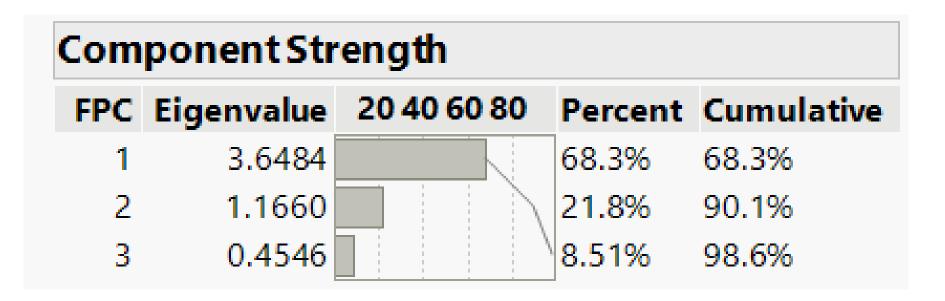
In general the Daubechies wavelets are chosen to have the highest number A of vanishing moments, (this does not imply the best smoothness) for given support width (number of coefficients) 2A.^[1] There are two naming schemes in use, DN using the length or number of taps, and dbA referring to the number of vanishing moments. So D4 and db2 are the same wavelet transform.

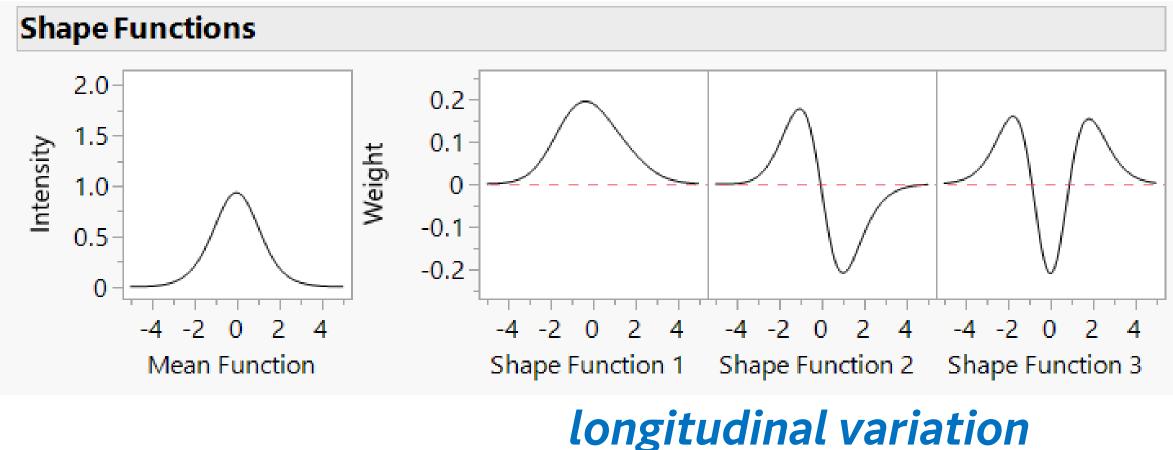
Model	Wavelet	AICc	BIC	GCV
1	Daubechies 20	-59532.1	-78876.4	3.44e-15
2	Symlet 20	-44379.4	-62187.8	1.35e-11
3	Symlet 10	-52644.6	-55665.3	5.25e-10
4	Daubechies 10	- <mark>5</mark> 2369.7	-55315.7	6.27e-10
5	Coiflet 5	-48479.4	-54470.6	8.06e-10
6	Daubechies 6	-39144.6	-40758.9	9.397e-7
7	Symlet 6	-39174.2	-40758.8	9.423e-7
8	Coiflet 3	-39355	-40067.7	1.448e-6
9	Symlet 4	-30273.7	-29262.9	0.000388
10	Coiflet 1	-19733.8	-16681.6	0.363082
11	Daubechies 2	-19678.2	-16665.7	0.358012
12	Haar	-11619.8	-8022.99	45.13531
13	Biorthogonal 1.3	-5971.84	-3170.2	257.5009
14	Biorthogonal 4.4	-2245.76	-1401.35	368.4419
15	Biorthogonal 2.6	11632.9	10392.76	96774.19



A wavelet is a wave like oscillation with an amplitude that begins at zero, increases or decreases, and then returns to zero one or more times. They are used to approximate more complex functions.



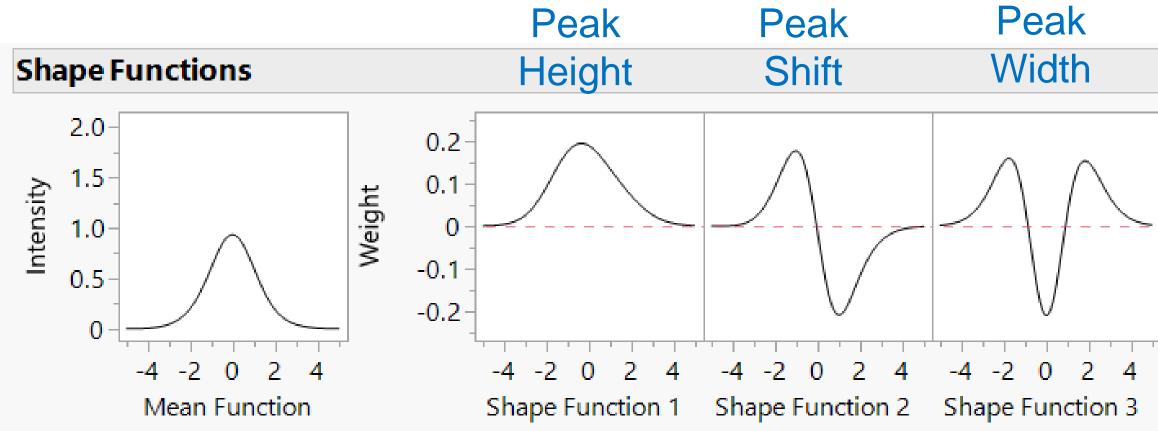




	Batch	FPC 1	FPC 2	FPC 3
1	1001	-1.08	-0.32	0.01
2	1002	1.39	1.77	-0.29
3	1003	1.02	-1.86	-0.44
4	1004	-1.42	-0.24	-0.48
5	1005	-1.20	-0.56	-0.91
6	1006	-0.97	-0.30	0.48
7	1007	- <mark>0.8</mark> 6	-1.13	0.54
8	1008	-2.33	-0.48	-0.22
9	1009	-1.61	-0.07	-0.15
10	1010	3.42	1.05	-0.76
11	1011	-1.53	<mark>0.9</mark> 9	-0.00
12	1012	3.78	-2.15	-0.10
13	1013	-0.91	1.57	-0.01
14	1014	2.80	<mark>0.8</mark> 8	-1.07
15	1015	1 <mark>.6</mark> 3	-0.21	0.40
16	1016	-0.07	1.45	0.64
17	1017	-0.75	<mark>0.5</mark> 5	0.67
18	1018	2.14	0.15	1.90
19	1019	-2.53	-0.08	-0.51
20	1020	-0.91	-1.03	0.29

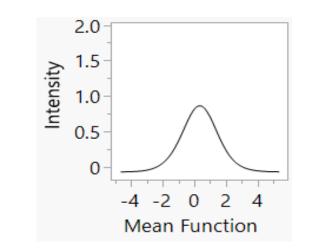
function-to-function variation

Component Strength FPC Eigenvalue 20406080 **Percent Cumulative** 3.6484 68.3% 68.3% 1 1.1660 21.8% 90.1% 2 0.4546 3 8.51% 98.6%

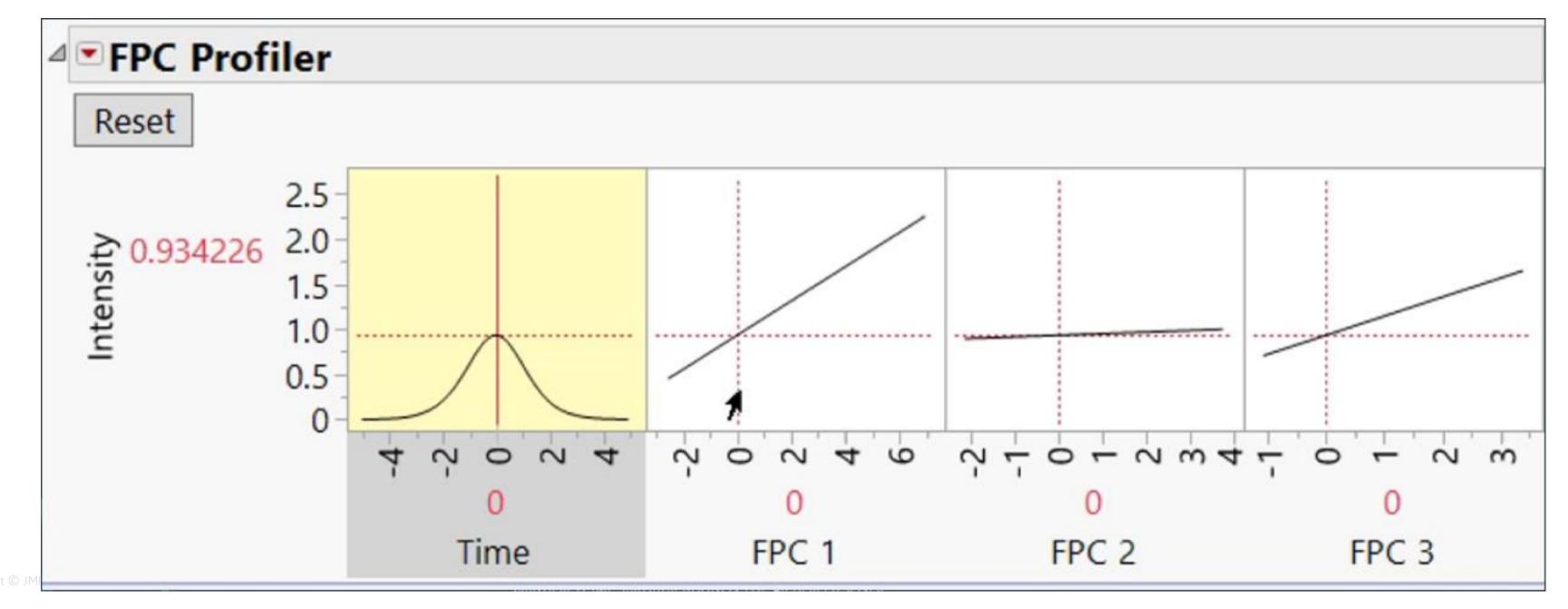


11001-1.08-0.320.01210021.391.77-0.29310031.02-1.86-0.4441004-1.42-0.24-0.4851005-1.20-0.56-0.9161006-0.97-0.300.4871007-0.86-1.130.5481008-2.33-0.48-0.2291009-1.61-0.07-0.151010103.421.05-0.76111011-1.530.99-0.001210123.78-2.15-0.10131013-0.911.57-0.011410142.800.88-1.071510151.63-0.210.40161016-0.071.450.64171017-0.750.550.671810182.140.151.90191019-2.53-0.08-0.51201020-0.91-1.030.29		Batch	FPC 1	FPC 2	FPC 3
3 1003 1.02 -1.86 -0.44 4 1004 -1.42 -0.24 -0.48 5 1005 -1.20 -0.56 -0.91 6 1006 -0.97 -0.30 0.48 7 1007 -0.86 -1.13 0.54 8 1008 -2.33 -0.48 -0.22 9 1009 -1.61 -0.07 -0.15 10 1010 3.42 1.05 -0.76 11 1011 -1.53 0.99 -0.00 12 1012 3.78 -2.15 -0.10 13 1013 -0.91 1.57 -0.01 14 1014 2.80 0.88 -1.07 15 1015 1.63 -0.21 0.40 16 1016 -0.07 1.45 0.64 17 1017 -0.75 0.55 0.67 18 1018 2.14 0.15 1.90	1	1001	-1.08	-0.32	0.01
4 1004 -1.42 -0.24 -0.48 5 1005 -1.20 -0.56 -0.91 6 1006 -0.97 -0.30 0.48 7 1007 -0.86 -1.13 0.54 8 1008 -2.33 -0.48 -0.22 9 1009 -1.61 -0.07 -0.15 10 1010 3.42 1.05 -0.76 11 1011 -1.53 0.99 -0.00 12 1012 3.78 -2.15 -0.10 13 1013 -0.91 1.57 -0.01 14 1014 2.80 0.88 -1.07 15 1015 1.63 -0.21 0.40 16 1016 -0.07 1.45 0.64 17 1017 -0.75 0.55 0.67 18 1018 2.14 0.15 1.90 19 1019 -2.53 -0.08 -0.51	2	1002	1.39	1.77	-0.29
5 1005 -1.20 -0.56 -0.91 6 1006 -0.97 -0.30 0.48 7 1007 -0.86 -1.13 0.54 8 1008 -2.33 -0.48 -0.22 9 1009 -1.61 -0.07 -0.15 10 1010 3.42 1.05 -0.76 11 1011 -1.53 0.99 -0.00 12 1012 3.78 -2.15 -0.10 13 1013 -0.91 1.57 -0.01 14 1014 2.80 0.88 -1.07 15 1015 1.63 -0.21 0.40 16 1016 -0.07 1.45 0.64 17 1017 -0.75 0.55 0.67 18 1018 2.14 0.15 1.90 19 1019 -2.53 -0.08 -0.51	3	1003	1.02	-1.86	-0.44
11000-0.97-0.300.4871007-0.86-1.130.5481008-2.33-0.48-0.2291009-1.61-0.07-0.151010103.421.05-0.76111011-1.530.99-0.001210123.78-2.15-0.10131013-0.911.57-0.011410142.800.88-1.071510151.63-0.210.40161016-0.071.450.64171017-0.750.550.671810182.140.151.90191019-2.53-0.08-0.51	4	1004	-1.42	-0.24	-0.48
7 1007 -0.86 -1.13 0.54 8 1008 -2.33 -0.48 -0.22 9 1009 -1.61 -0.07 -0.15 10 1010 3.42 1.05 -0.76 11 1011 -1.53 0.99 -0.00 12 1012 3.78 -2.15 -0.10 13 1013 -0.91 1.57 -0.01 14 1014 2.80 0.88 -1.07 15 1015 1.63 -0.21 0.40 16 1016 -0.07 1.45 0.64 17 1017 -0.75 0.55 0.67 18 1018 2.14 0.15 1.90 19 1019 -2.53 -0.08 -0.51	5	1005	-1.20	-0.56	-0.91
8 1008 -2.33 -0.48 -0.22 9 1009 -1.61 -0.07 -0.15 10 1010 3.42 1.05 -0.76 11 1011 -1.53 0.99 -0.00 12 1012 3.78 -2.15 -0.10 13 1013 -0.91 1.57 -0.01 14 1014 2.80 0.88 -1.07 15 1015 1.63 -0.21 0.40 16 1016 -0.07 1.45 0.64 17 1017 -0.75 0.55 0.67 18 1018 2.14 0.15 1.90 19 1019 -2.53 -0.08 -0.51	6	1006	-0.97	-0.30	0.48
91009-1.61-0.07-0.151010103.421.05-0.76111011-1.530.99-0.001210123.78-2.15-0.10131013-0.911.57-0.011410142.800.88-1.071510151.63-0.210.40161016-0.071.450.64171017-0.750.550.671810182.140.151.90191019-2.53-0.08-0.51	7	1007	-0.86	-1.13	0.54
1010103.421.010.011010103.421.05-0.76111011-1.530.99-0.001210123.78-2.15-0.10131013-0.911.57-0.011410142.800.88-1.071510151.63-0.210.40161016-0.071.450.64171017-0.750.550.671810182.140.151.90191019-2.53-0.08-0.51	8	1008	-2.33	-0.48	-0.22
111011-1.530.99-0.001210123.78-2.15-0.10131013-0.911.57-0.011410142.800.88-1.071510151.63-0.210.40161016-0.071.450.64171017-0.750.550.671810182.140.151.90191019-2.53-0.08-0.51	9	1009	-1.61	-0.07	-0.15
12 1012 3.78 -2.15 -0.10 13 1013 -0.91 1.57 -0.01 14 1014 2.80 0.88 -1.07 15 1015 1.63 -0.21 0.40 16 1016 -0.07 1.45 0.64 17 1017 -0.75 0.55 0.67 18 1018 2.14 0.15 1.90 19 1019 -2.53 -0.08 -0.51	10	1010	3.42	1.05	-0.76
131013-0.911.57-0.011410142.800.88-1.071510151.63-0.210.40161016-0.071.450.64171017-0.750.550.671810182.140.151.90191019-2.53-0.08-0.51	11	1011	-1.53	<mark>0.9</mark> 9	-0.00
1410142.800.88-1.071510151.63-0.210.40161016-0.071.450.64171017-0.750.550.671810182.140.151.90191019-2.53-0.08-0.51	12	1012	3.78	-2.15	-0.10
1510151.63-0.210.40161016-0.071.450.64171017-0.750.550.671810182.140.151.90191019-2.53-0.08-0.51	13	1013	-0.91	1.57	-0.01
161016-0.071.450.64171017-0.750.550.671810182.140.151.90191019-2.53-0.08-0.51	14	1014	2.80	<mark>0.8</mark> 8	-1.07
17 1017 -0.75 0.55 0.67 18 1018 2.14 0.15 1.90 19 1019 -2.53 -0.08 -0.51	15	1015	1.63	-0.21	0.40
18 1018 2.14 0.15 1.90 19 1019 -2.53 -0.08 -0.51	16	1016	-0.07	1.45	0.64
19 1019 -2.53 -0.08 -0.51	17	1017	-0.75	0.55	0.67
	18	1018	2.14	0.15	1.90
20 1020 -0.91 -1.03 0.29	19	1019	-2.53	-0.08	-0.51
	20	1020	-0.91	-1.03	0.29



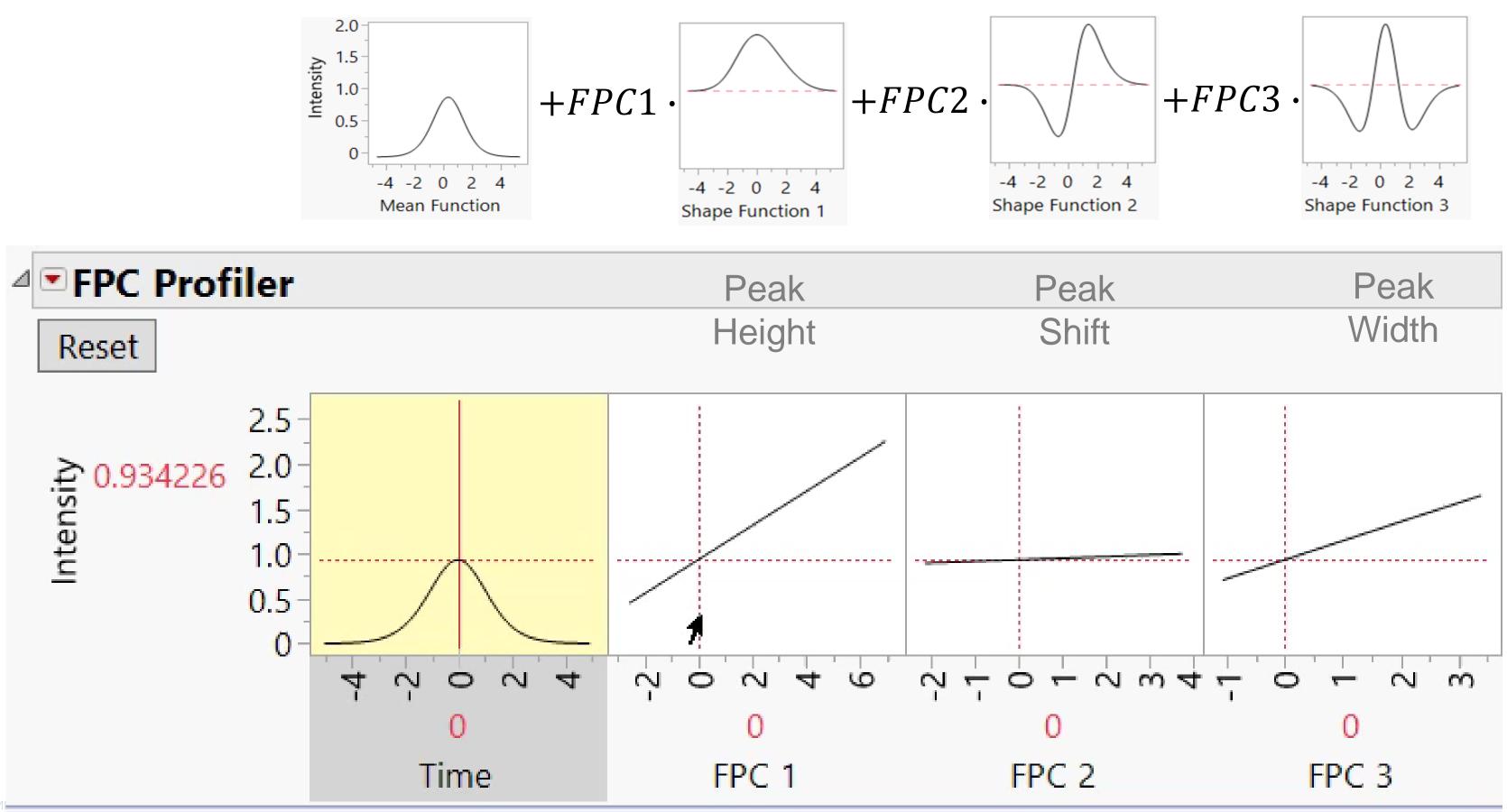


When all zero





FPC Scores \rightarrow Mean Curve





Outline

- Why / What DOE?
- What is Functional Data Analysis (FDA)?
- Reverse Engineering Case #1 Modeling alcohol blends
- Reverse Engineer Case #2 Mineral formulations

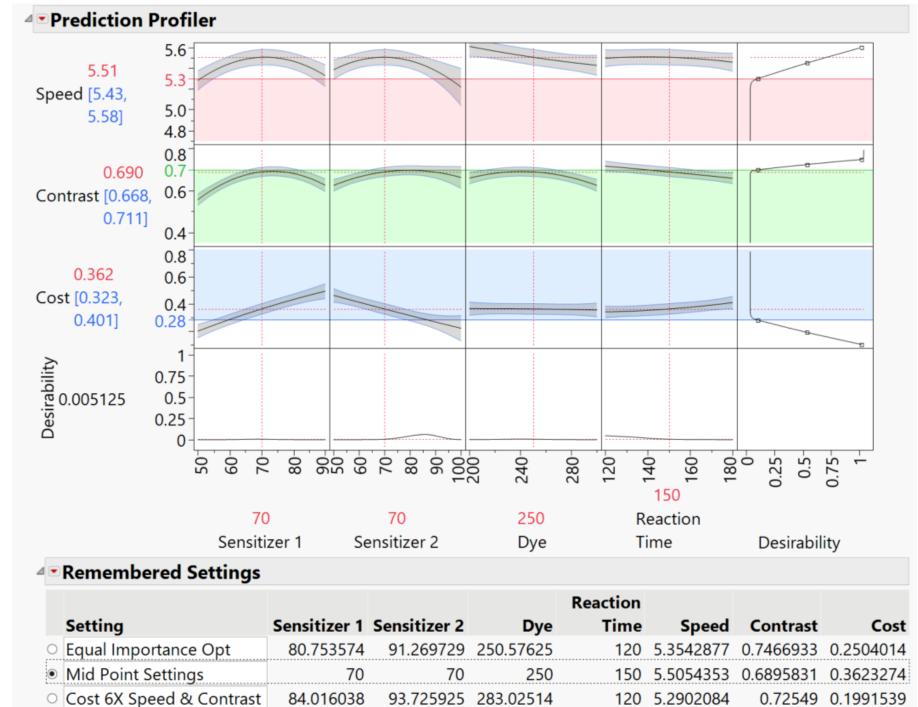




There's no better way to get the most information from the least amount of data than to use Design of Experiments methods

Opt Spd3X-Cntr1X-Cost6X

90 90 300 180 5.59 0.73 1 50 50 200 120 4.79 0.36 2 50 50 250 120 5.36 0.616 0 2 50 50 200 180 5.39 0.537 0 3 90 70 200 120 5.31 0.623 0 4 50 90 200 150 5.13 0.431 0 5 70 70 250 180 5.37 0.643 0 6 50 90 300 120 4.79 0.375 0 7 90 90 200 180 5.45 0.626 0 8 90 50 250 150 5.00 0.470 0 9 50 50 300 150 5.22 0.478 0 10 70 90 <	⊿ 7/4 €	Sensitizer	Sensitizer		Reaction			
27/0 50 50 200 120 4.79 0.36 1 50 50 250 120 5.36 0.616 0 2 50 50 200 180 5.39 0.537 0 3 90 70 200 120 5.31 0.623 0 4 50 90 200 150 5.13 0.431 0 5 70 70 250 180 5.37 0.643 0 6 50 90 300 120 4.79 0.375 0 7 90 90 200 180 5.45 0.626 0 8 90 50 250 150 5.00 0.470 0 9 50 50 300 150 5.22 0.478 0 10 70 90 200 120 5.33 0.734 0 11		1	2	Dye	Time	Speed	Contrast	Cost
1 50 50 250 120 5.36 0.616 0 2 50 50 200 180 5.39 0.537 0 3 90 70 200 120 5.31 0.623 0 4 50 90 200 150 5.13 0.431 0 5 70 70 250 180 5.37 0.643 0 6 50 90 300 120 4.79 0.375 0 7 90 90 200 180 5.45 0.626 0 8 90 50 250 150 5.00 0.470 0 9 50 50 300 150 5.22 0.478 0 10 70 90 200 120 5.33 0.734 0 11 90 90 250 120 5.32 0.577 0 <		90	90	300	180	5.59	0.73	0.73
1 50 50 250 120 5.36 0.616 0 2 50 50 200 180 5.39 0.537 0 3 90 70 200 120 5.31 0.623 0 4 50 90 200 150 5.13 0.431 0 5 70 70 250 180 5.37 0.643 0 6 50 90 300 120 4.79 0.375 0 7 90 90 200 180 5.45 0.626 0 8 90 50 250 150 5.00 0.470 0 9 50 50 300 150 5.22 0.478 0 10 70 90 200 120 5.33 0.734 0 11 90 90 250 120 5.32 0.577 0 <								
1 50 50 250 120 5.36 0.616 0 2 50 50 200 180 5.39 0.537 0 3 90 70 200 120 5.31 0.623 0 4 50 90 200 150 5.13 0.431 0 5 70 70 250 180 5.37 0.643 0 6 50 90 300 120 4.79 0.375 0 7 90 90 200 180 5.45 0.626 0 8 90 50 250 150 5.00 0.470 0 9 50 50 300 150 5.22 0.478 0 10 70 90 200 120 5.33 0.734 0 11 90 90 250 120 5.32 0.577 0 <		50	50	200	100	4.70	0.25	0.17
2 50 50 200 180 5.39 0.537 0 3 90 70 200 120 5.31 0.623 0 4 50 90 200 150 5.33 0.431 0 5 70 70 250 180 5.37 0.643 0 6 50 90 300 120 4.79 0.375 0 7 90 90 200 180 5.45 0.626 0 8 90 50 250 150 5.00 0.470 0 9 50 50 300 150 5.22 0.478 0 10 70 90 200 120 5.41 0.668 0 11 90 90 250 120 5.33 0.734 0 13 70 50 200 150 5.47 0.688 0								0.17
3 90 70 200 120 5.31 0.623 0 4 50 90 200 150 5.13 0.431 0 5 70 70 250 180 5.37 0.643 0 6 50 90 300 120 4.79 0.375 0 7 90 90 200 180 5.45 0.626 0 8 90 50 250 150 5.00 0.470 0 9 50 50 300 150 5.22 0.478 0 10 70 90 200 120 5.41 0.668 0 11 90 90 250 120 5.33 0.774 0 12 50 50 200 150 5.47 0.688 0 13 70 70 250 150 5.47 0.688 0	-							0.175
4 50 90 200 150 5.13 0.431 0 5 70 70 250 180 5.37 0.643 0 6 50 90 300 120 4.79 0.375 0 7 90 90 200 180 5.45 0.626 0 8 90 50 250 150 5.00 0.470 0 9 50 50 300 150 5.22 0.478 0 10 70 90 200 120 5.33 0.734 0 11 90 90 250 120 5.32 0.574 0 12 50 50 200 150 5.43 0.596 0 13 70 70 250 180 5.57 0.689 0 14 50 70 250 150 5.43 0.518 0.518								0.447
5 70 70 250 180 5.37 0.643 0 6 50 90 300 120 4.79 0.375 0 7 90 90 200 180 5.45 0.626 0 8 90 50 250 150 5.00 0.470 0 9 50 50 300 150 5.22 0.478 0 10 70 90 200 120 5.41 0.668 0 11 90 90 250 120 5.33 0.734 0 12 50 50 250 120 5.32 0.574 0 13 70 50 200 150 5.43 0.589 0 14 50 70 250 180 5.57 0.689 0 15 70 70 250 150 5.47 0.688 0								0.177
6 50 90 300 120 4.79 0.375 0 7 90 90 200 180 5.45 0.626 0 8 90 50 250 150 5.00 0.470 0 9 50 50 300 150 5.22 0.478 0 10 70 90 200 120 5.33 0.734 0 11 90 90 250 120 5.33 0.734 0 12 50 50 250 120 5.33 0.754 0 13 70 50 200 150 5.49 0.596 0 14 50 70 250 180 5.27 0.689 0 15 70 70 250 150 5.47 0.688 0 16 90 90 300 150 5.47 0.688 0								0.445
7 90 90 200 180 5.45 0.626 0 8 90 50 250 150 5.00 0.470 0 9 50 50 300 150 5.22 0.478 0 10 70 90 200 120 5.41 0.668 0 11 90 90 250 120 5.33 0.734 0 12 50 50 250 120 5.32 0.574 0 13 70 50 200 150 5.49 0.596 0 14 50 70 250 180 5.22 0.583 0 15 70 70 250 150 5.57 0.689 0 16 90 90 300 150 5.42 0.657 0 16 90 90 300 150 5.43 0.518 0								0.231
950503001505.220.47801070902001205.410.66801190902501205.330.73401250502501205.320.57401370502001505.490.59601450702501805.220.55801570702501805.220.65301690903001505.470.68801690903001505.420.65701690903001505.420.65801770702501505.430.51801870702001205.430.51802050503001505.150.50502190702001205.330.66102250903001205.090.49202390503001805.030.35802490503001805.590.70702670903001805.590.70702670903001805.550.6050								0.471
10 70 90 200 120 5.41 0.668 0 11 90 90 250 120 5.33 0.734 0 12 50 50 250 120 5.32 0.574 0 13 70 50 200 150 5.49 0.596 0 14 50 70 250 180 5.22 0.558 0 14 50 70 250 150 5.57 0.689 0 15 70 70 250 150 5.47 0.688 0 16 90 90 300 150 5.47 0.688 0 17 70 70 250 150 5.47 0.688 0 18 70 70 200 120 5.43 0.518 0 20 50 70 200 120 5.43 0.518 0 21 90 70 200 120 5.43 0.515 0.505 0<	8	90	50	250	150	5.00	0.470	0.670
11 90 90 250 120 5.33 0.734 0 12 50 50 250 120 5.32 0.574 0 13 70 50 200 150 5.49 0.596 0 14 50 70 250 180 5.22 0.558 0 15 70 70 250 150 5.57 0.689 0 16 90 90 300 150 5.47 0.688 0 17 70 70 250 150 5.47 0.688 0 18 70 70 250 150 5.47 0.688 0 19 50 70 200 120 5.43 0.518 0 20 50 50 300 120 5.43 0.518 0 21 90 70 200 120 5.33 0.661 0	9	50	50	300	150	5.22	0.478	0.283
12 50 50 250 120 5.32 0.574 0 13 70 50 200 150 5.49 0.596 0 14 50 70 250 180 5.22 0.558 0 15 70 70 250 150 5.57 0.689 0 16 90 90 300 150 5.26 0.653 0 17 70 70 250 150 5.47 0.688 0 18 70 70 200 120 5.42 0.657 0 19 50 70 200 120 5.43 0.518 0 20 50 50 300 150 5.15 0.505 0 21 90 70 200 120 5.33 0.661 0 22 50 90 300 120 5.09 0.492 0 22 50 90 50 300 180 5.03 0.358 0 <td>10</td> <td>70</td> <td>90</td> <td>200</td> <td>120</td> <td>5.41</td> <td>0.668</td> <td>0.226</td>	10	70	90	200	120	5.41	0.668	0.226
13 70 50 200 150 5.49 0.596 0 14 50 70 250 180 5.22 0.558 0 15 70 70 250 150 5.57 0.689 0 16 90 90 300 150 5.26 0.653 0 17 70 70 250 150 5.47 0.688 0 17 70 70 250 150 5.47 0.688 0 18 70 70 300 120 5.42 0.657 0 19 50 70 200 120 5.43 0.518 0 20 50 50 300 150 5.15 0.505 0 21 90 70 200 120 5.33 0.661 0 22 50 90 300 120 5.09 0.411 0 22 50 90 50 300 180 5.03 0.358 0 <td>11</td> <td>90</td> <td>90</td> <td>250</td> <td>120</td> <td>5.33</td> <td>0.734</td> <td>0.310</td>	11	90	90	250	120	5.33	0.734	0.310
14 50 70 250 180 5.22 0.558 0 15 70 70 250 150 5.57 0.689 0 16 90 90 300 150 5.26 0.653 0 17 70 70 250 150 5.47 0.688 0 18 70 70 250 150 5.47 0.688 0 18 70 70 300 120 5.42 0.657 0 19 50 70 200 120 5.43 0.518 0 20 50 50 300 150 5.15 0.505 0 21 90 70 200 120 5.33 0.661 0 22 50 90 300 120 5.99 0.411 0 223 90 50 300 120 5.09 0.492 0 24 90 50 300 180 5.03 0.358 0 <	12	50	50	250	120	5.32	0.574	0.257
15 70 70 250 150 5.57 0.689 0 16 90 90 300 150 5.26 0.653 0 17 70 70 250 150 5.47 0.688 0 18 70 70 250 150 5.42 0.657 0 19 50 70 300 120 5.43 0.518 0 20 50 50 300 150 5.43 0.518 0 210 90 50 50 300 150 5.15 0.505 0 22 50 90 300 120 5.33 0.661 0 22 50 90 300 120 4.97 0.411 0 223 90 50 300 180 5.03 0.358 0 24 90 50 300 180 5.59 0.707 0 <td>13</td> <td>70</td> <td>50</td> <td>200</td> <td>150</td> <td>5.49</td> <td>0.596</td> <td>0.456</td>	13	70	50	200	150	5.49	0.596	0.456
16 90 90 300 150 5.26 0.653 0 17 70 70 250 150 5.47 0.688 0 18 70 70 300 120 5.42 0.657 0 19 50 70 200 120 5.43 0.518 0 20 50 50 300 150 5.15 0.505 0 20 50 50 300 150 5.15 0.505 0 21 90 70 200 120 5.33 0.661 0 22 50 90 300 120 5.09 0.411 0 23 90 50 300 120 5.09 0.492 0 24 90 50 300 180 5.03 0.358 0 25 70 70 250 150 5.59 0.707 0 2	14	50	70	250	180	5.22	0.558	0.166
17770702501505.470.68801870703001205.420.65701950702001205.430.51802005005003001505.150.50502190702001205.330.66102250903001204.970.41102390503001205.090.49202490503001805.030.35802570702501505.590.70702670903001805.250.6050	15	70	70	250	150	5.57	0.689	0.390
18 70 70 300 120 5.42 0.657 0 19 50 70 200 120 5.43 0.518 0 20 50 50 300 150 5.15 0.505 0 21 90 70 200 120 5.33 0.661 0 22 50 90 300 120 5.33 0.661 0 22 50 90 300 120 4.97 0.411 0 23 90 50 300 120 5.09 0.492 0 24 90 50 300 180 5.03 0.358 0 25 70 70 250 150 5.59 0.707 0 26 70 90 300 180 5.25 0.605 0	16	90	90	300	150	5.26	0.653	0.226
19 50 70 200 120 5.43 0.518 0 20 50 50 300 150 5.15 0.505 0 21 90 70 200 120 5.33 0.661 0 22 50 90 300 120 4.97 0.411 0 23 90 50 300 120 5.09 0.492 0 24 90 50 300 120 5.03 0.358 0 24 90 50 300 180 5.03 0.358 0 25 70 70 250 150 5.59 0.707 0 26 70 90 300 180 5.25 0.605 0	17	70	70	250	150	5.47	0.688	0.356
20 50 50 300 150 5.15 0.505 0 21 90 70 200 120 5.33 0.661 0 22 50 90 300 120 4.97 0.411 0 23 90 50 300 120 5.09 0.492 0 24 90 50 300 180 5.03 0.358 0 25 70 70 250 150 5.59 0.707 0 26 70 90 300 180 5.25 0.605 0	18	70	70	300	120	5.42	0.657	0.337
21 90 70 200 120 5.33 0.661 0 22 50 90 300 120 4.97 0.411 0 23 90 50 300 120 5.09 0.492 0 24 90 50 300 180 5.03 0.358 0 25 70 70 250 150 5.59 0.707 0 26 70 90 300 180 5.25 0.605 0	19	50	70	200	120	5.43	0.518	0.222
22 50 90 300 120 4.97 0.411 0 23 90 50 300 120 5.09 0.492 0 24 90 50 300 180 5.03 0.358 0 25 70 70 250 150 5.59 0.707 0 26 70 90 300 180 5.25 0.605 0	20	50	50	300	150	5.15	0.505	0.287
23 90 50 300 120 5.09 0.492 0 24 90 50 300 180 5.03 0.358 0 25 70 70 250 150 5.59 0.707 0 26 70 90 300 180 5.25 0.605 0	21	90	70	200	120	5.33	0.661	0.457
24 90 50 300 180 5.03 0.358 0 25 70 70 250 150 5.59 0.707 0 26 70 90 300 180 5.25 0.605 0	22	50	90	300	120	4.97	0.411	0.191
25 70 70 250 150 5.59 0.707 0 26 70 90 300 180 5.25 0.605 0	23	90	50	300	120	5.09	0.492	0.588
26 70 90 300 180 5.25 0.605 0	24	90	50	300	180	5.03	0.358	0.733
	25	70	70	250	150	5.59	0.707	0.318
27 50 90 200 150 5.24 0.476 0	26	70	90	300	180		0.605	0.290
	27	50	90	200	150	5.24	0.476	0.177

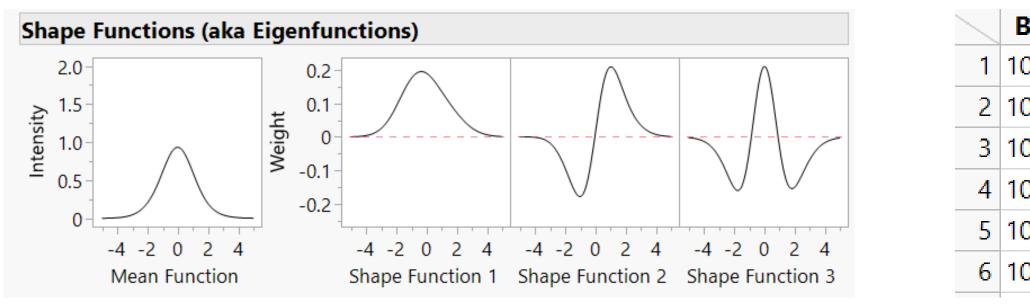


			Reaction				
Sensitizer 1	Sensitizer 2	Dye	Time	Speed	Contrast	Cost	
80.753574	91.269729	250.57625	120	5.3542877	0.7466933	0.2504014	
70	70	250	150	5.5054353	0.6895831	0.3623274	
84.016038	93.725925	283.02514	120	5.2902084	0.72549	0.1991539	
81.958309	90.706277	286.82246	120	5.3269582	0.7177857	0.2211116	



Takeaway #2

- FDA breaks apart highly correlated longitudinal data like spectra into two parts:
 - 1. Shape functions explaining the longitudinal variation
 - 2. FPC Scores explaining the function-to-function variation



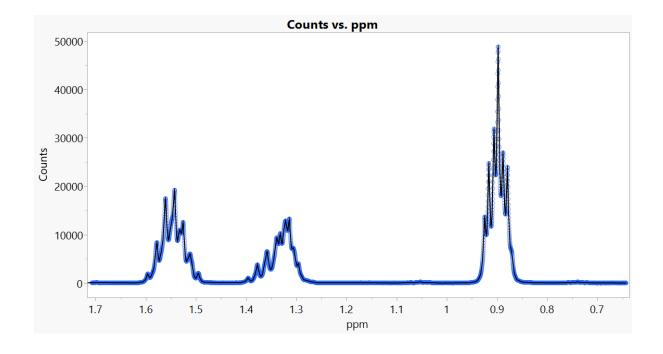
Batch	FPC 1	FPC 2	FPC 3
001	-1.08	-0.32	0.01
002	1.39	1.77	-0.29
003	1.02	-1.86	-0.44
004	-1.42	-0.24	-0.48
005	-1.20	-0.56	-0.91
006	-0.97	-0.30	0.48



Takeaway #3

• FDA using wavelets can be combined with mixture DOE analysis to build models that predict spectra as a function of formulation component proportions.

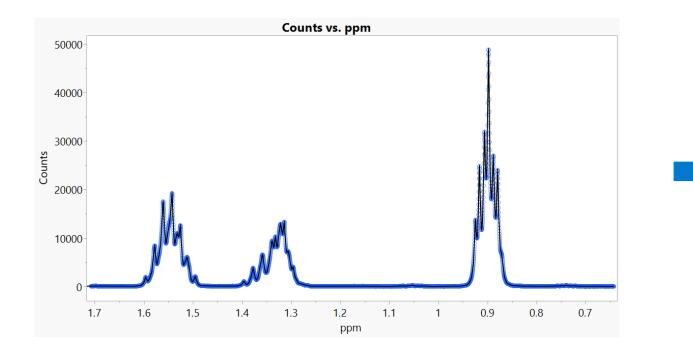




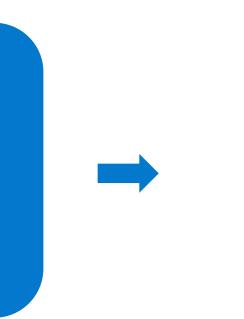


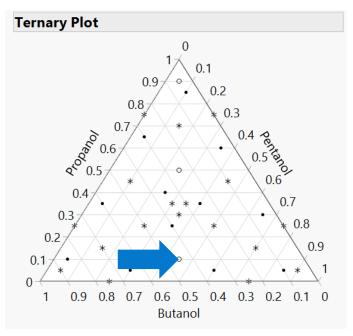


•Using spectra for an unknown blend as a "target" and an FDA-DOE mixture model, one can predict formulation component proportions. **Reverse Engineer it!**



FDA-DOE Model

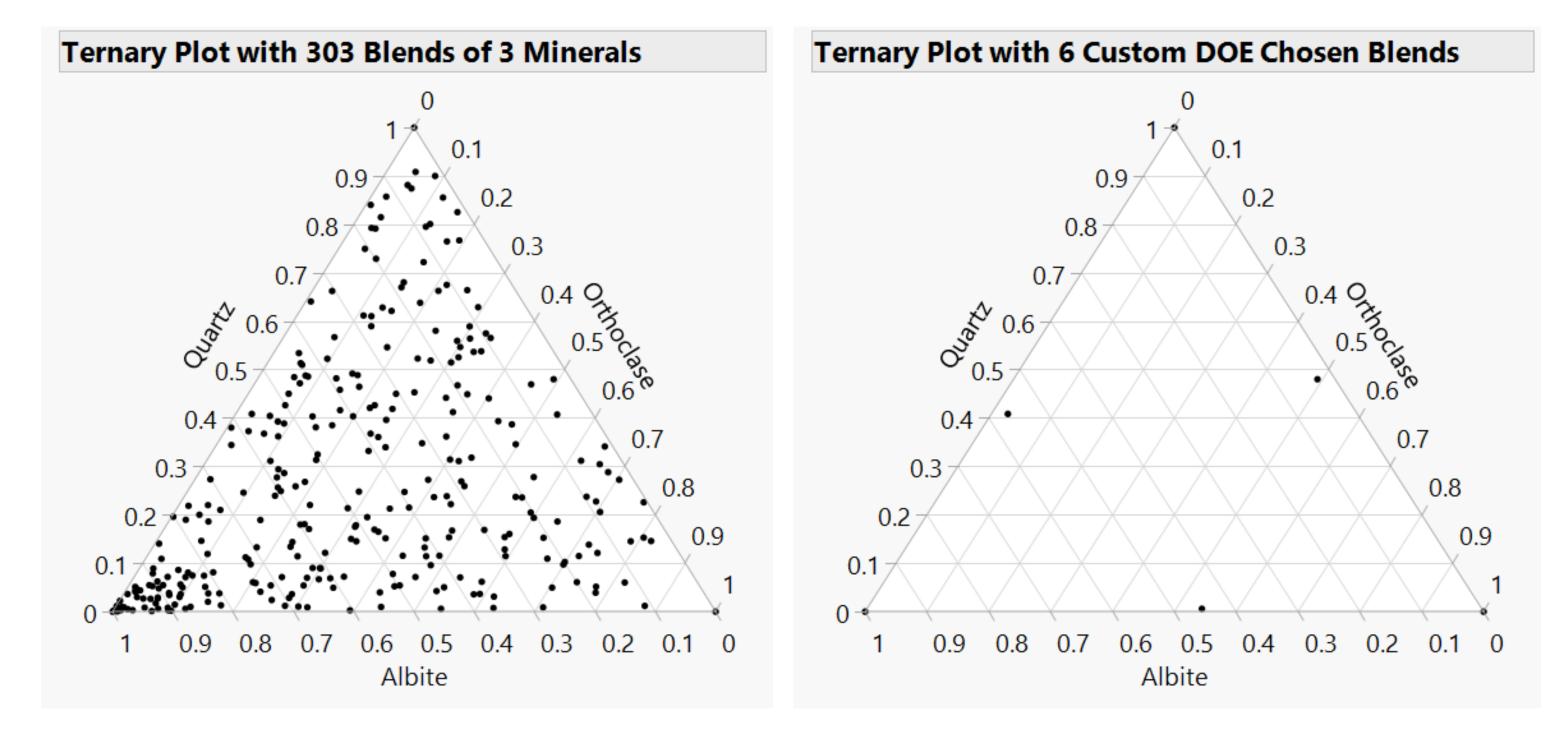






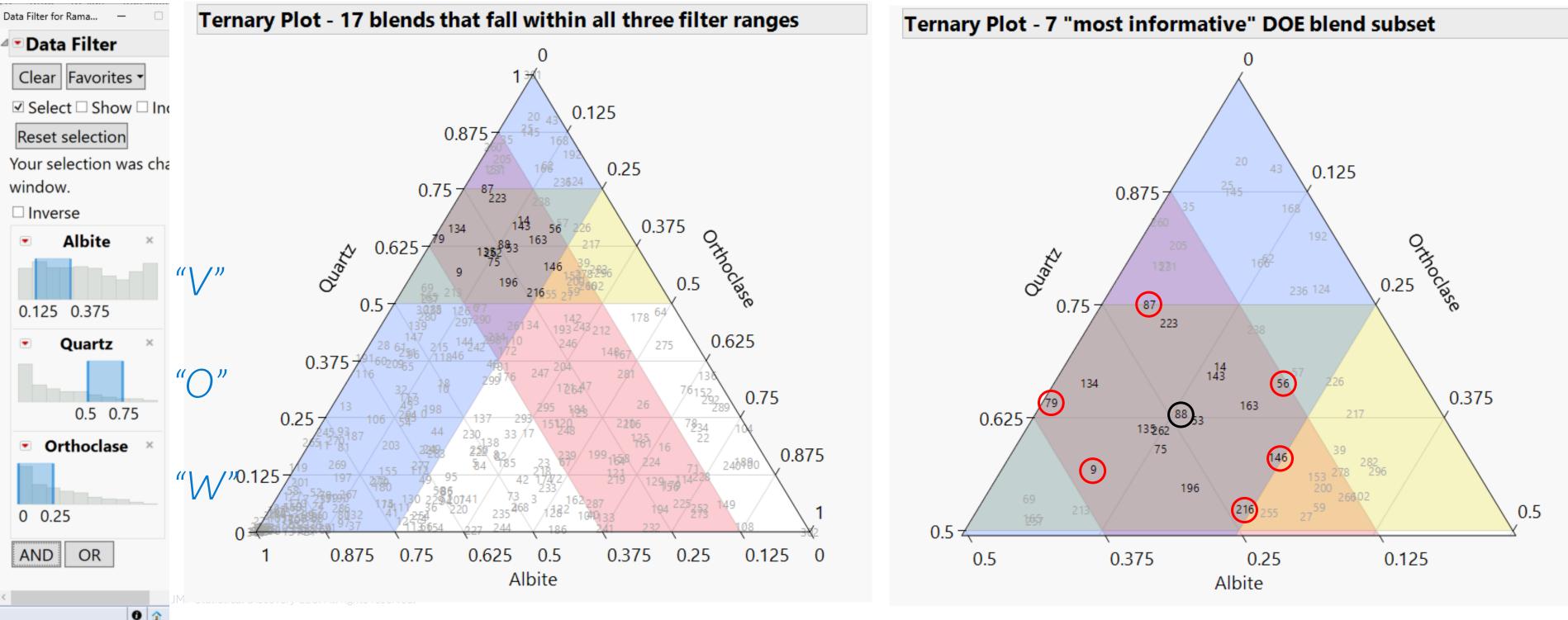


• Use Custom DOE to choose the most informative subset of trials (blends) from existing trials used as a candidate set.





Takeaway #5 (continued) Use Custom DOE on a constrained region of design space 17 *filtered* blends used as candidate trials 7 circled trials are DOE subset



Case 1: Reanalysis of NMR Spectral Data for 3-Alcohol Mixture DOE using Functional Data Analysis



Available online at www.sciencedirect.com



Journal of Magnetic Resonance 190 (2008) 26-32

Quantitative analysis of NMR spectra with chemometrics

H. Winning *, F.H. Larsen, R. Bro, S.B. Engelsen

Quality and Technology, Department of Food Science, Faculty of Life Sciences, University of Copenhagen, Rolighedsvej 30, DK-1958 Frederiksberg C, Denmark

> Received 11 April 2007; revised 8 October 2007 Available online 14 October 2007

ſ

JMR Journal of Magnetic Resonance

www.elsevier.com/locate/jmr



Quantitative analysis of NMR spectra with chemometrics

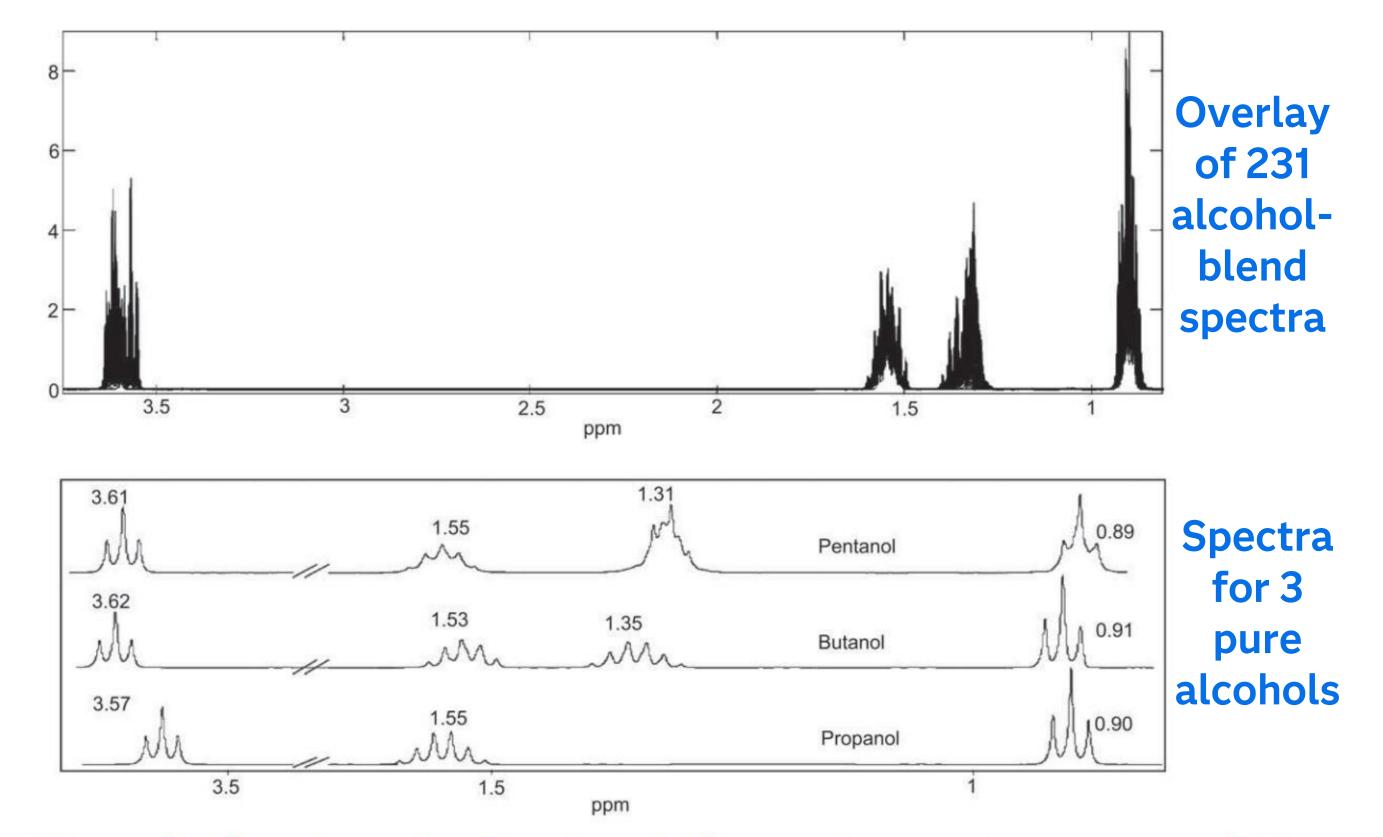


Fig. 2. (Top) NMR spectra of the 231 alcohol mixtures from 3.85 to 0.65 ppm. The NMR spectra of mixtures show highly overlapping signals. (Bottom) The ¹H NMR spectra of the pure alcohol samples of propanol, butanol and pentanol.



Quantitative analysis of NMR spectra with chemometrics

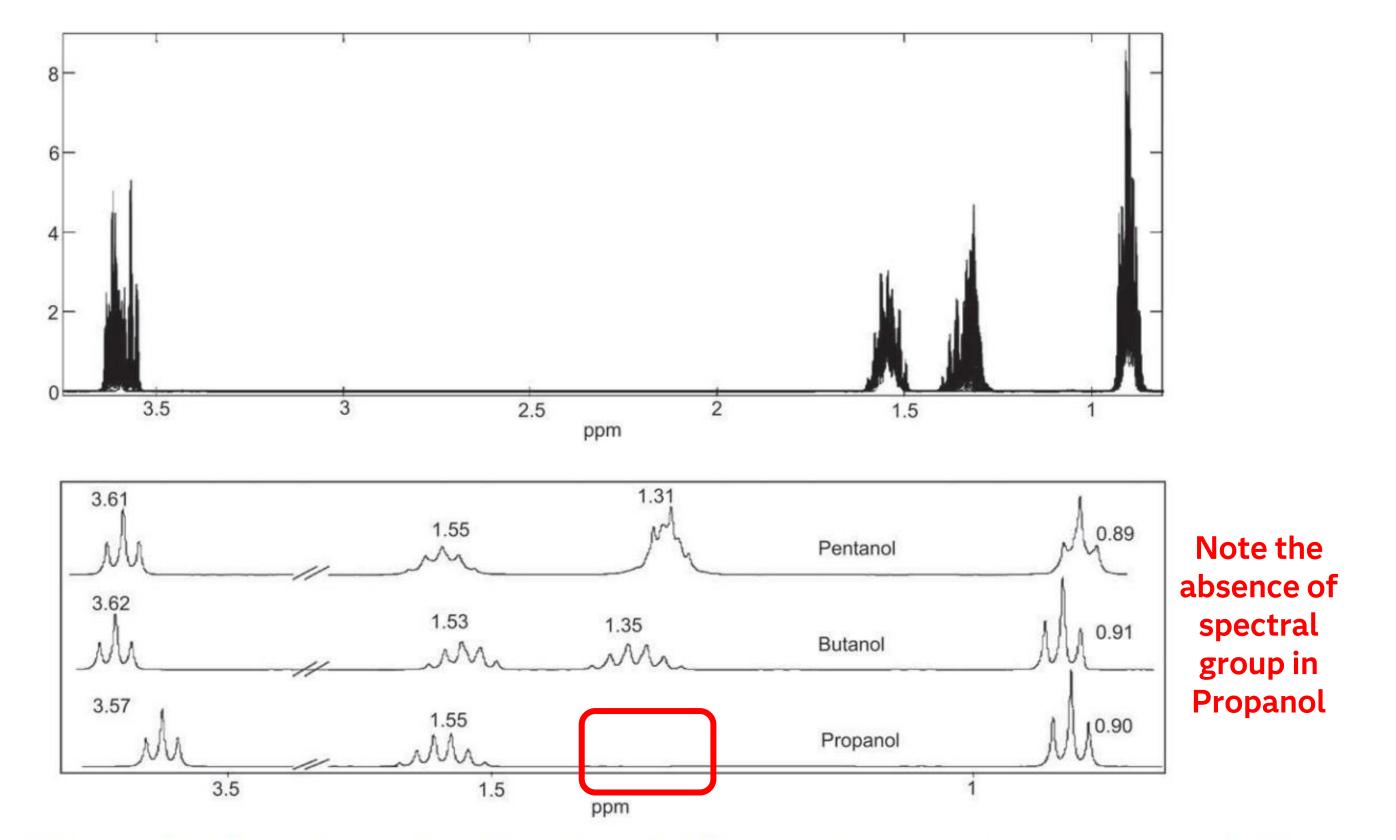
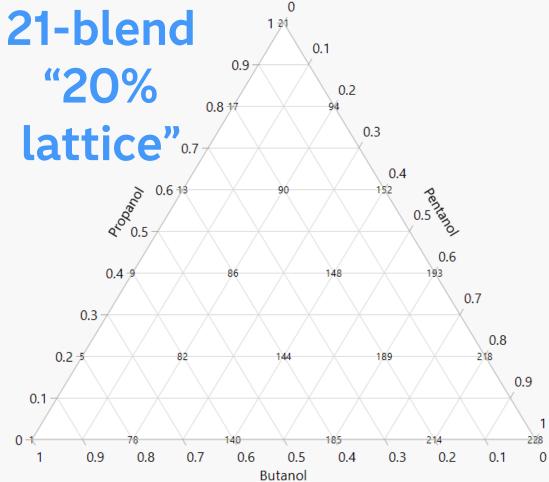


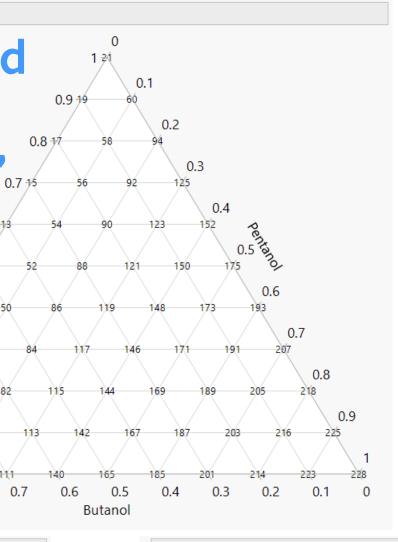
Fig. 2. (Top) NMR spectra of the 231 alcohol mixtures from 3.85 to 0.65 ppm. The NMR spectra of mixtures show highly overlapping signals. (Bottom) The ¹H NMR spectra of the pure alcohol samples of propanol, butanol and pentanol.

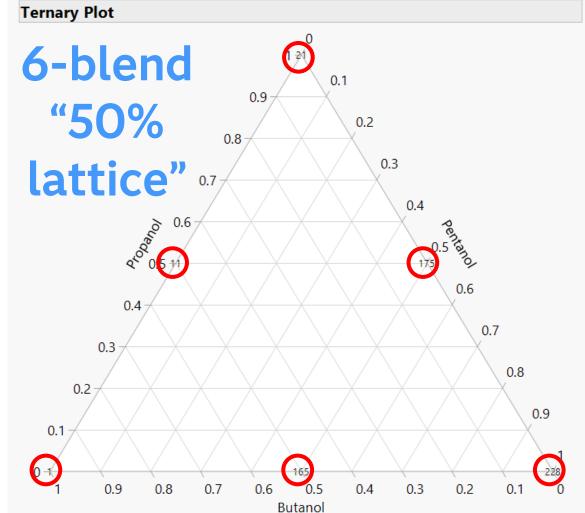




Various size "lattice" subset mixture designs



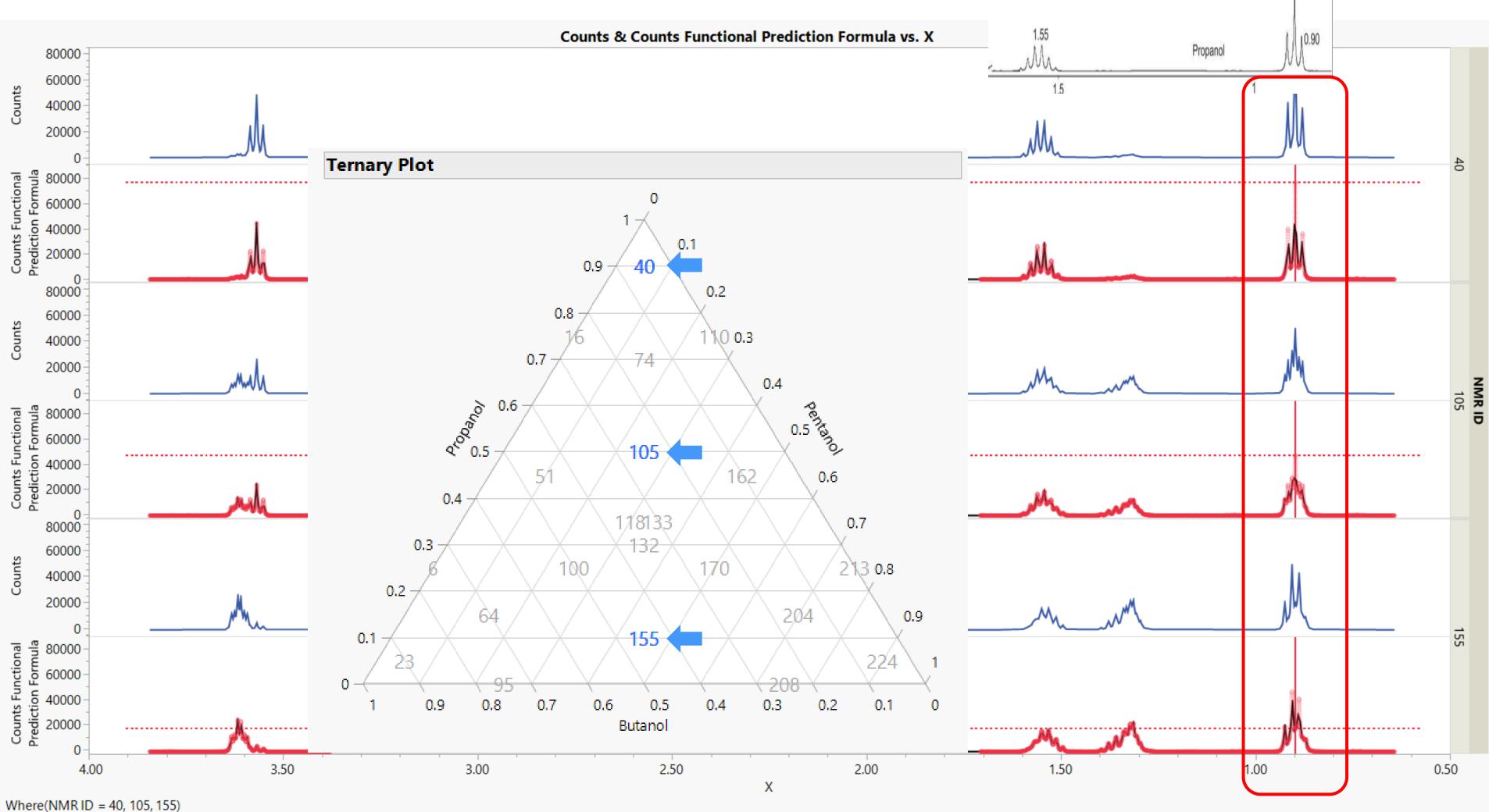


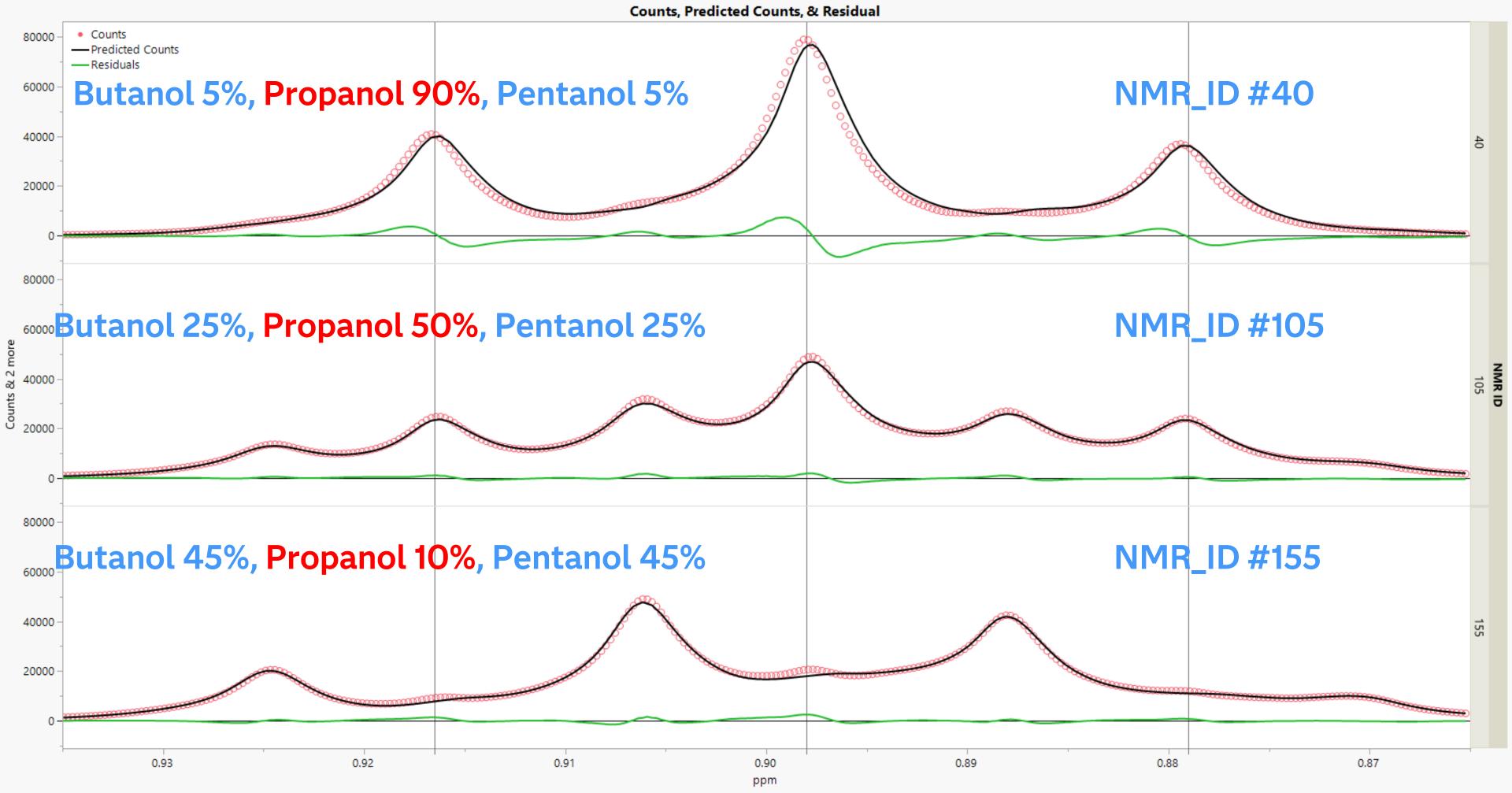


Given proportions of each component in a formulation - that was not used in the functional data analysis -

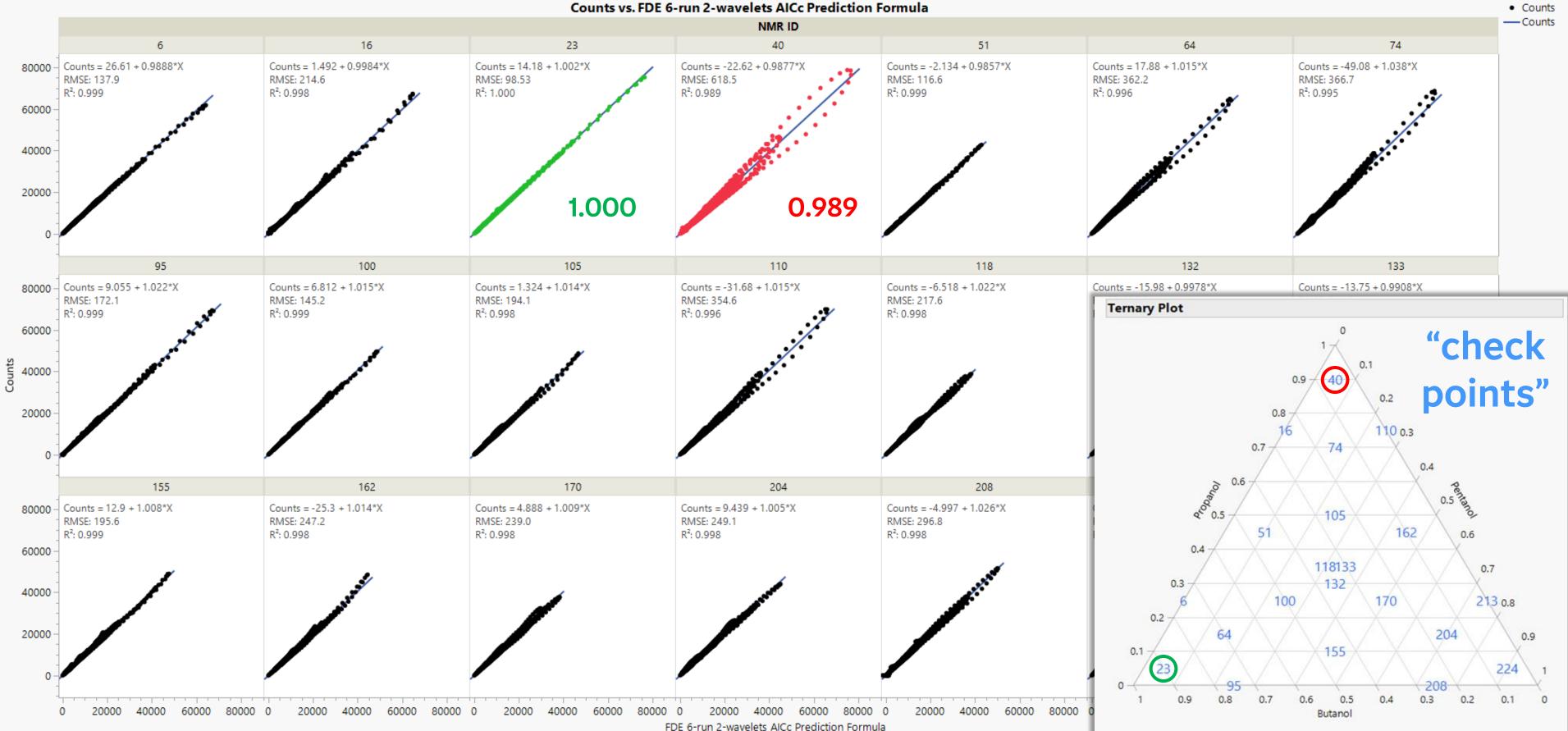
How well can the spectral shape be predicted?





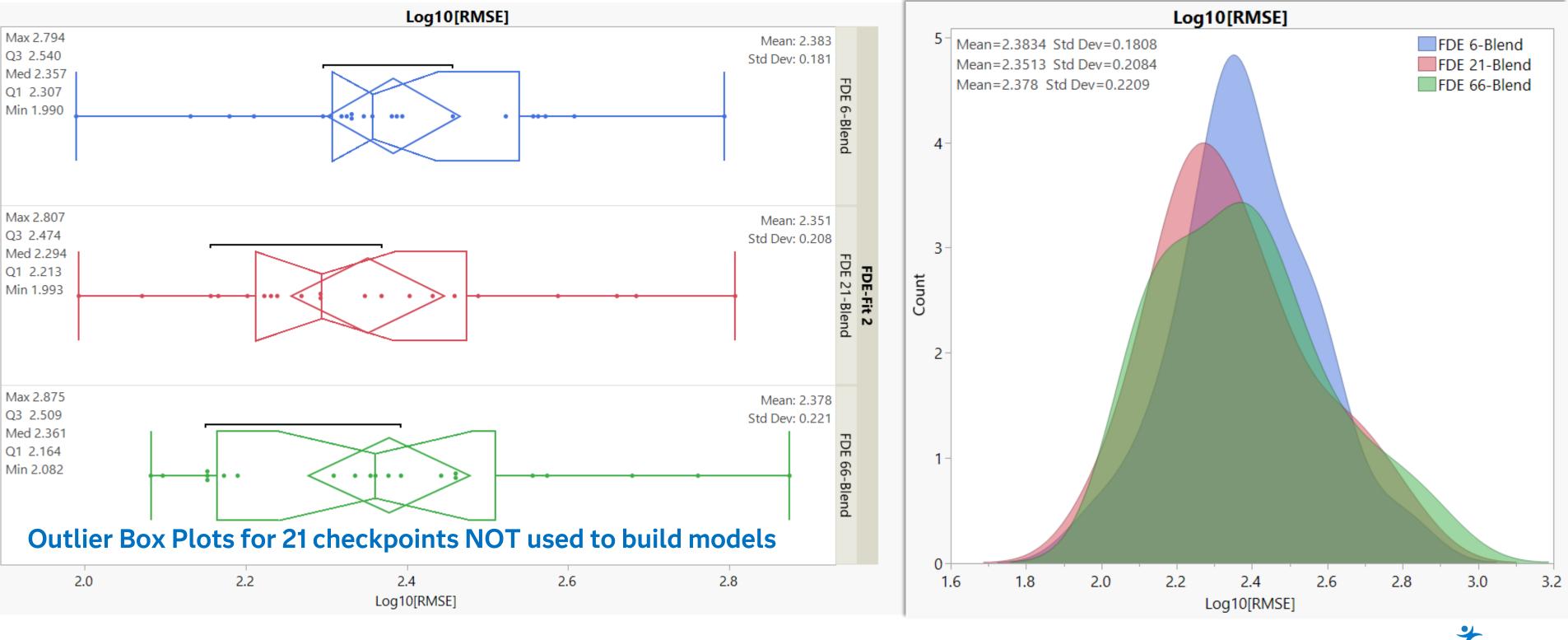


Actual vs. Predicted Plots for 21 Test Blends



FDE 6-run 2-wavelets AICc Prediction Formula

No Statistical or Practical Improvement Observed when Increasing from 6 to 21 or 66 Blends to Train FDA Model

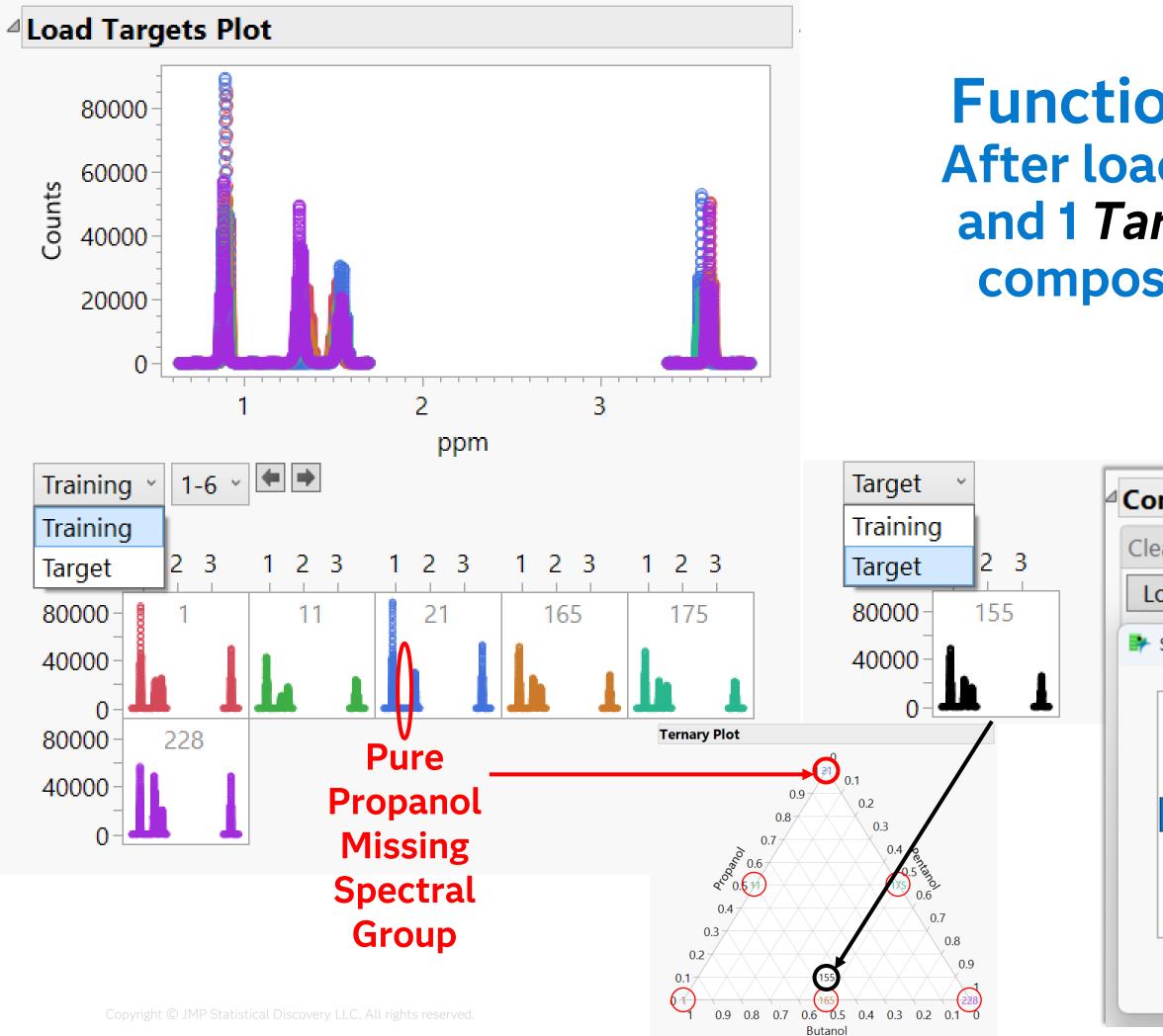




Given the spectral shape of a formulation - that was not used in the functional data analysis -

How well can the proportion of each component **be predicted? Can we Reverse Engineer it?**

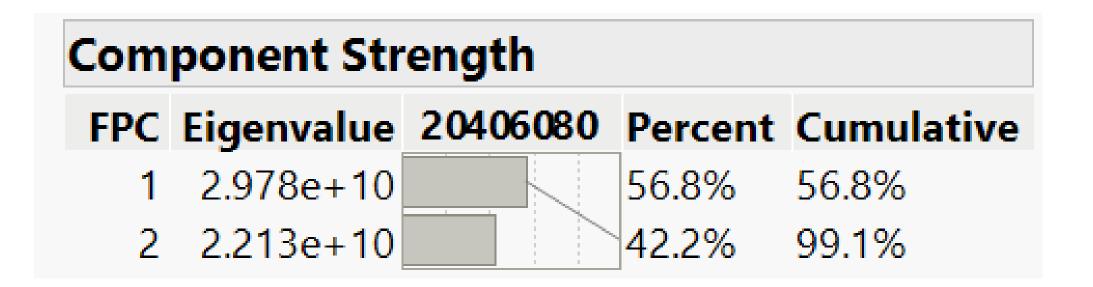


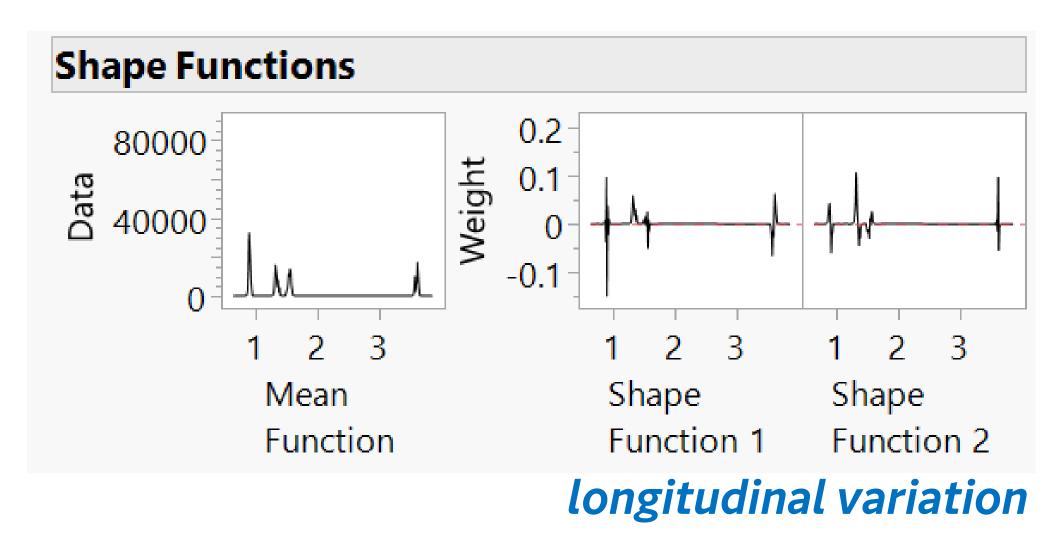


Functional Data Explorer: After loading 6 *Training* spectra and 1 *Target* spectra to predict composition of NMR ID #155

					_
mma	nds				
anup	Transform	Align	Spectral	Target Function	s
bad					
Select target functions				×	
1					
11					
21					
155					
165					
175					
228					
			ОК	Cancel	

FUNCTIONAL DATA ANALYSIS







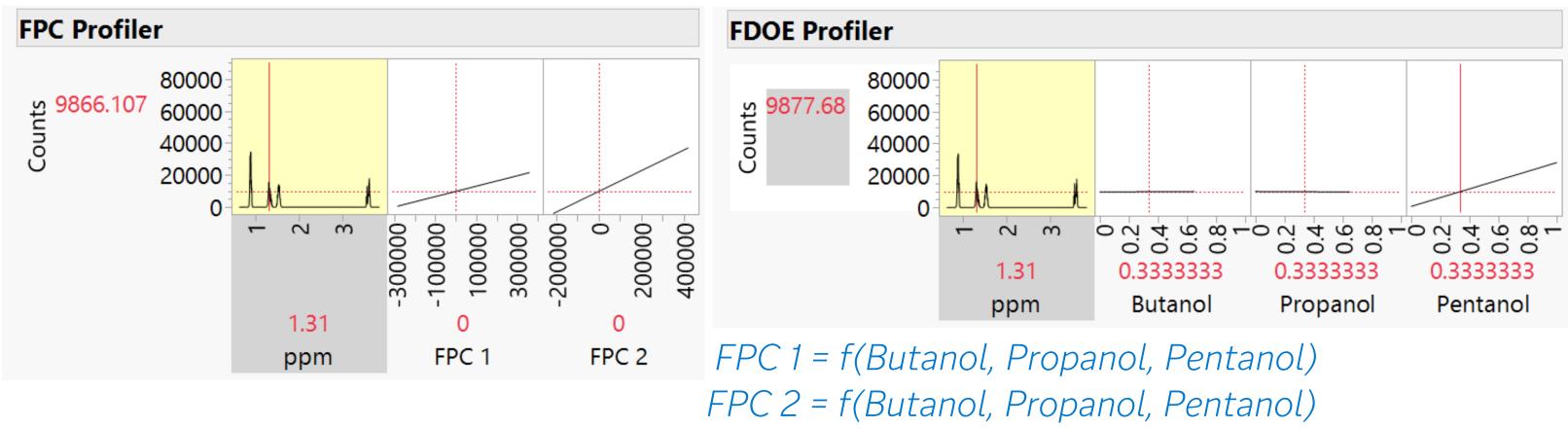
Function Summaries						
NMR ID	FPC 1	FPC 2				
1	145028.69	-213039.7				
11	-73207.5	-101250.7				
21	-283858.6	- <mark>5560.768</mark>				
165	140129.43	3883.9851				
175	-63377.54	107639.32				
228	135285.5	208327.83				

function-to-function variation



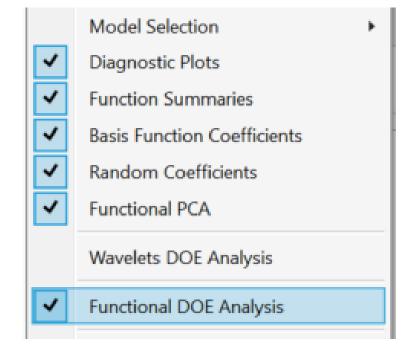
FUNCTIONAL DOE

- **One Problem!!** We don't blend FPCs. We blend alcohols!
- That is why Functional DOE Analysis option is so powerful
- Automatically models the FPC scores as functions of the DOE factors



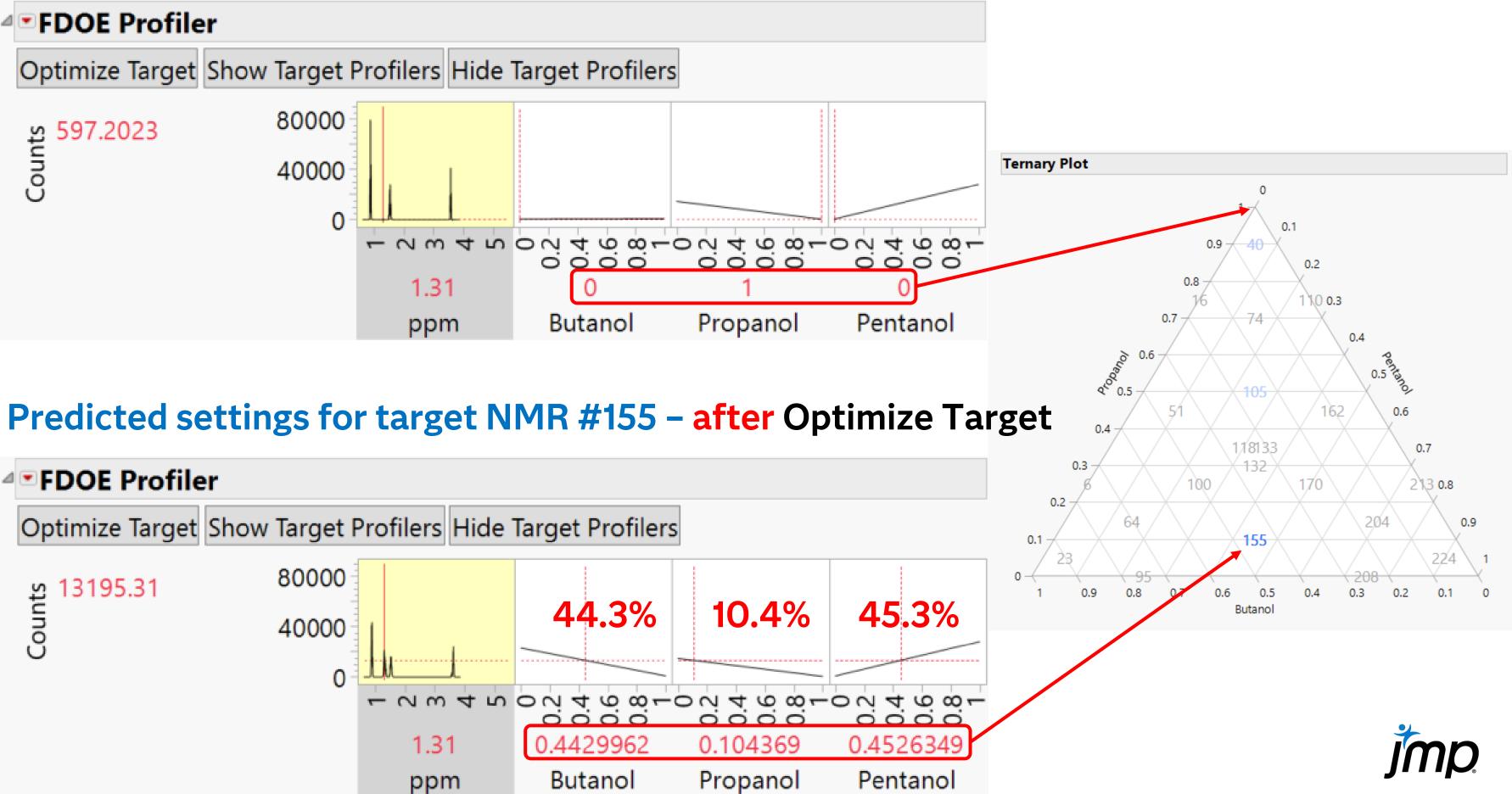
• FDOE Profiler model can readily be used - in a practical manner to *predict* component/factor impact on spectra

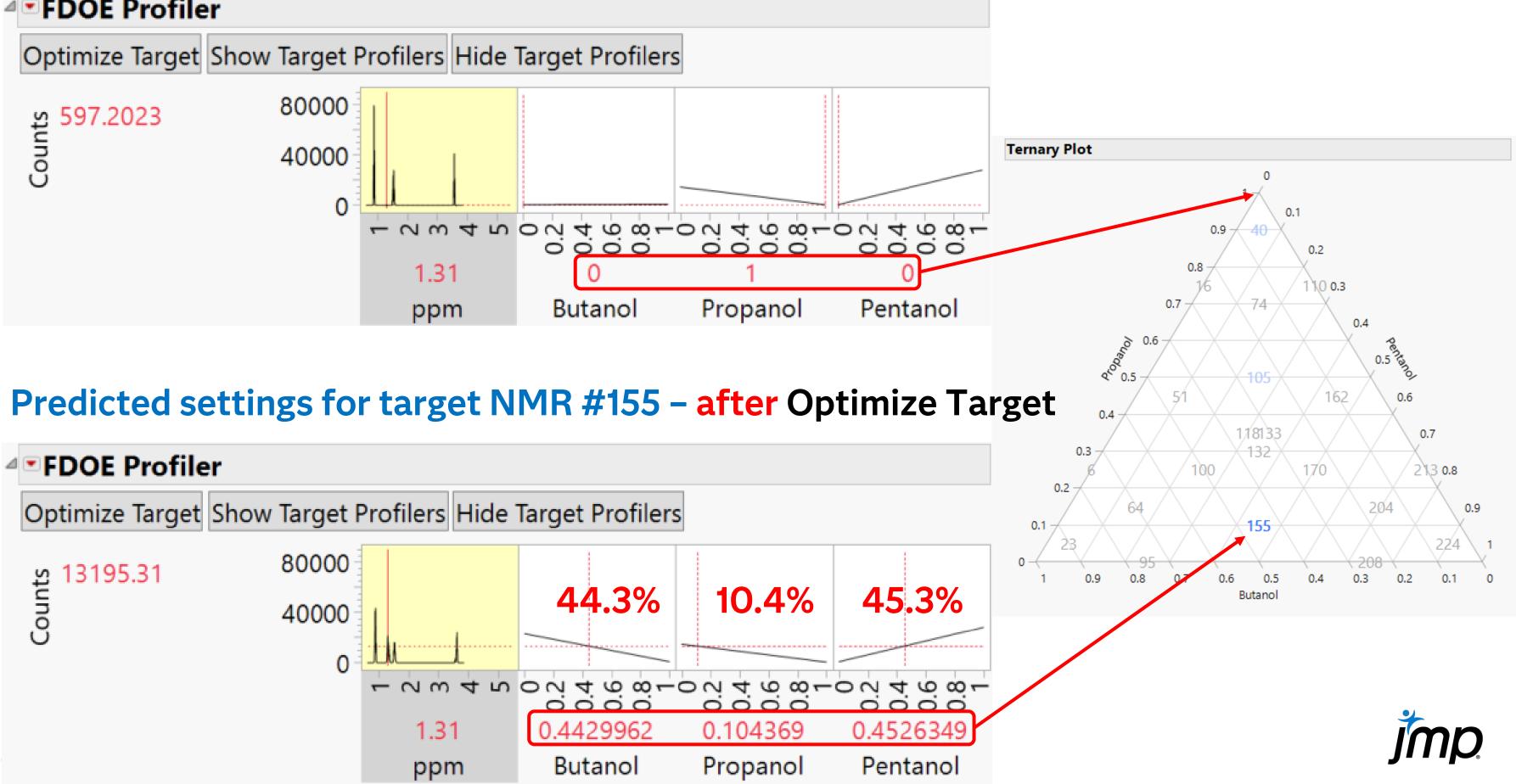




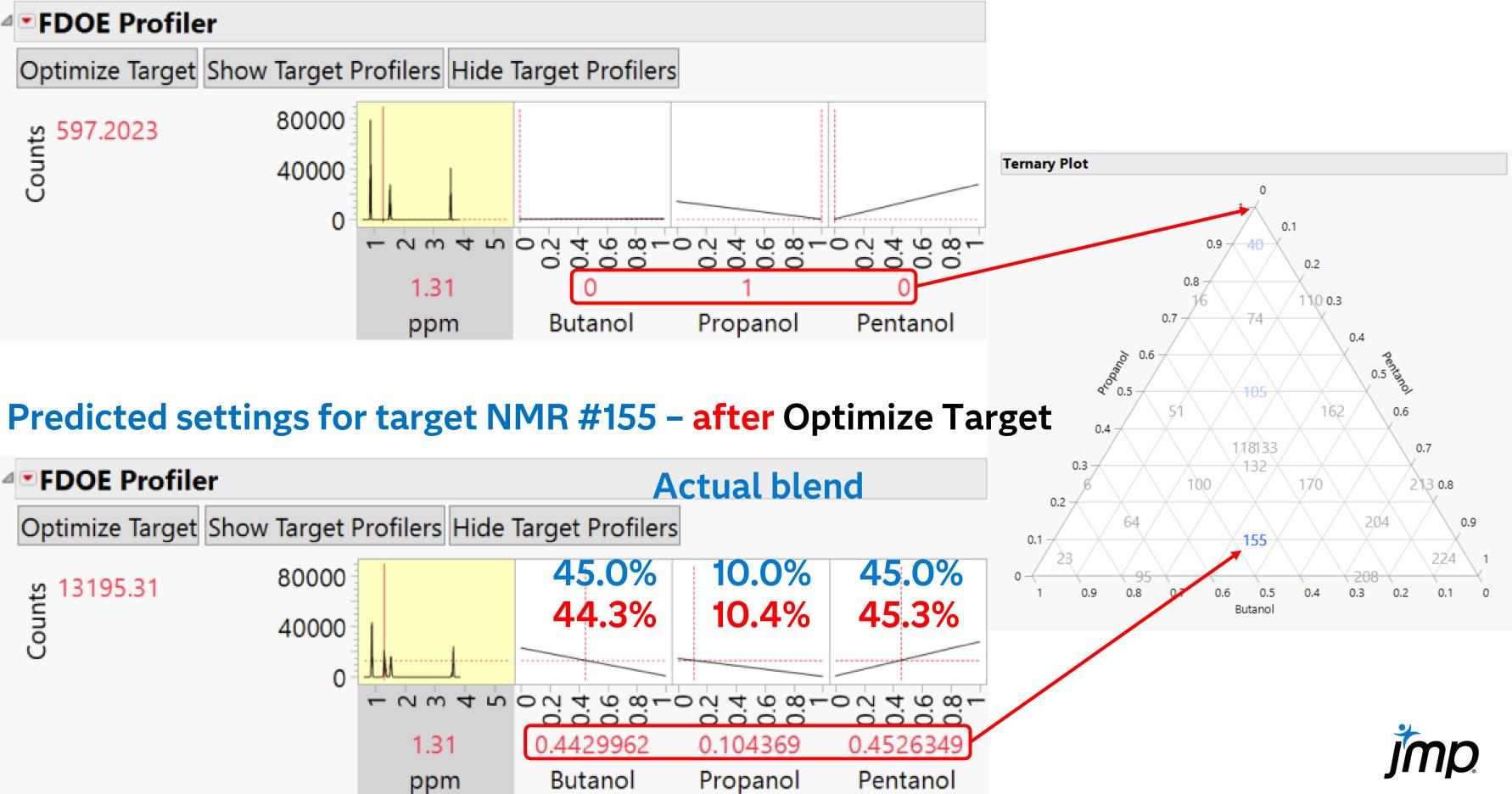


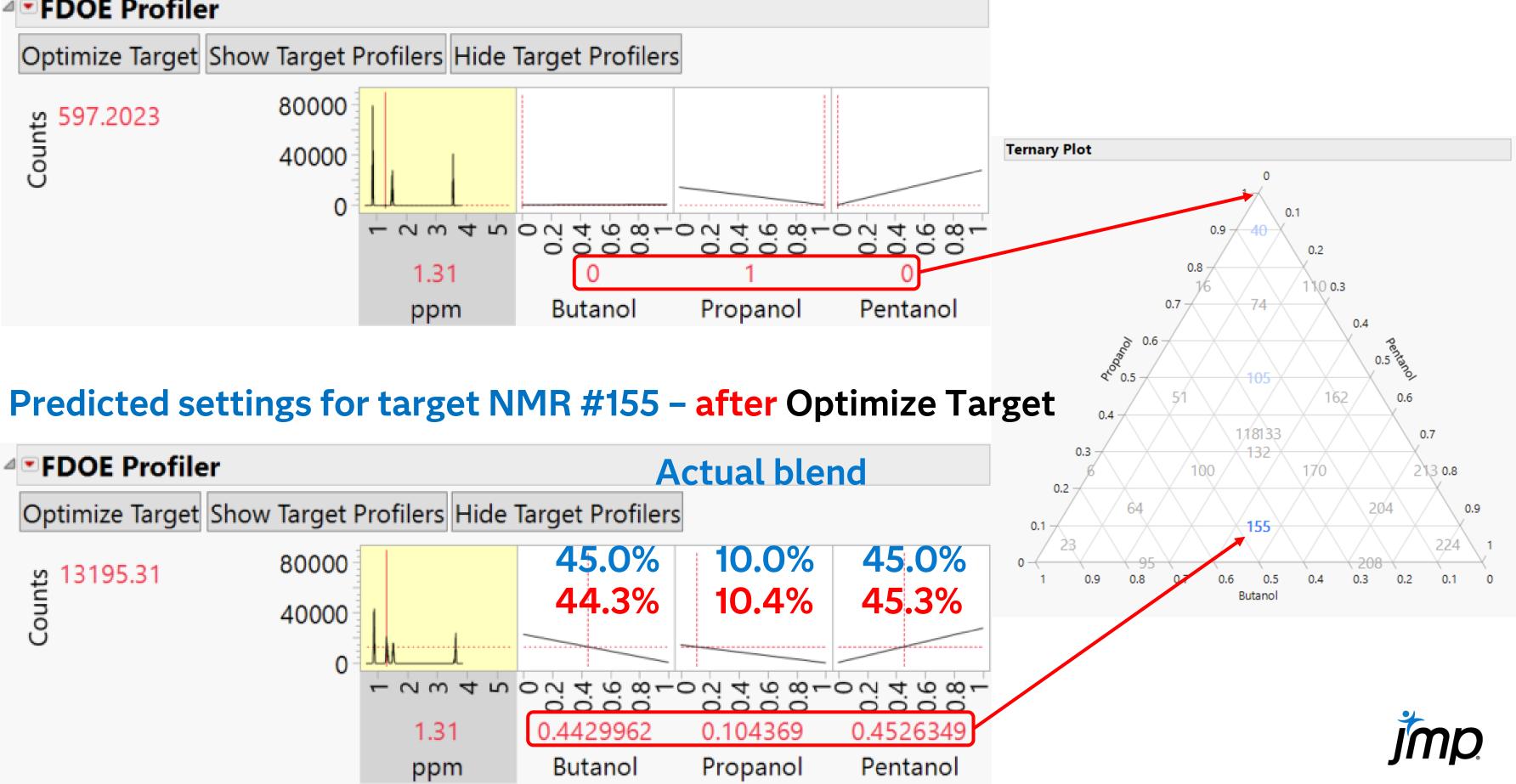
Settings for pure Propanol (0% 100% 0%) – before Optimize Target

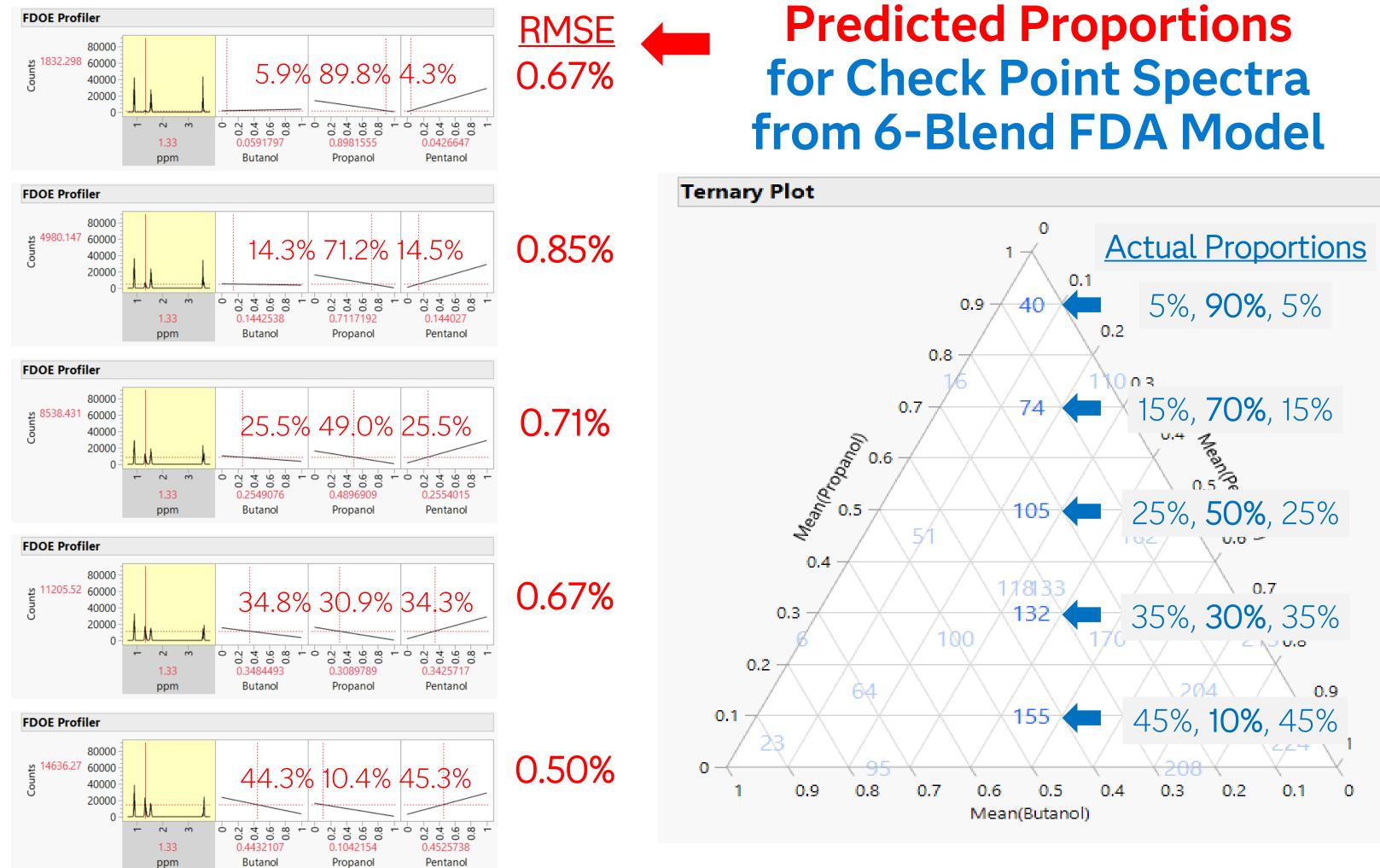




Settings for pure Propanol (0% 100% 0%) – before Optimize Target









Let's go to JMP...

• Perform FDA on the NMR spectra of 6 alcohol blends and identify the composition of the 7th target blend



How might this be used in industries like chemical, biopharma, food, & consumer products

- Run a DOE with component and factor ranges you believe 1. encompass the unknown competitor formulation settings
- 2. Use FDA to model the spectra of the DOE blends & conditions
- Use FDA-DOE model and the target spectra of competitor's 3. product to closely determine the actual formulation



How might this be used in industries like defense and aerospace?

Help formulate chemical composition of a decoy flare, 1. so its spectra matches that of a particular aircraft engine

2. By analyzing

- spectra of sensors
- sonar signatures
- radar signatures

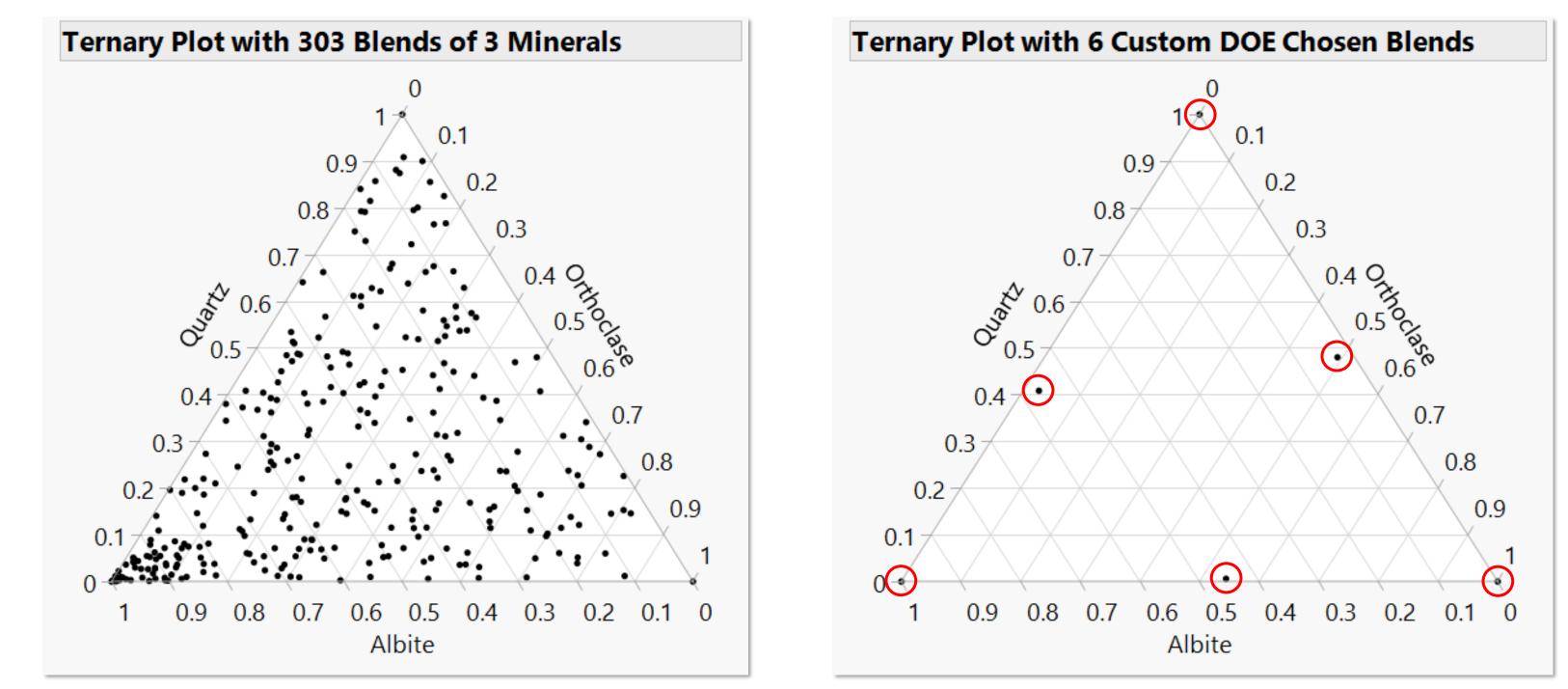
All as functions of factors like shape, speed, angle, distance, vibration, etc.

Better predict remaining useful life of engines and improve identification of detected submersibles and aircraft

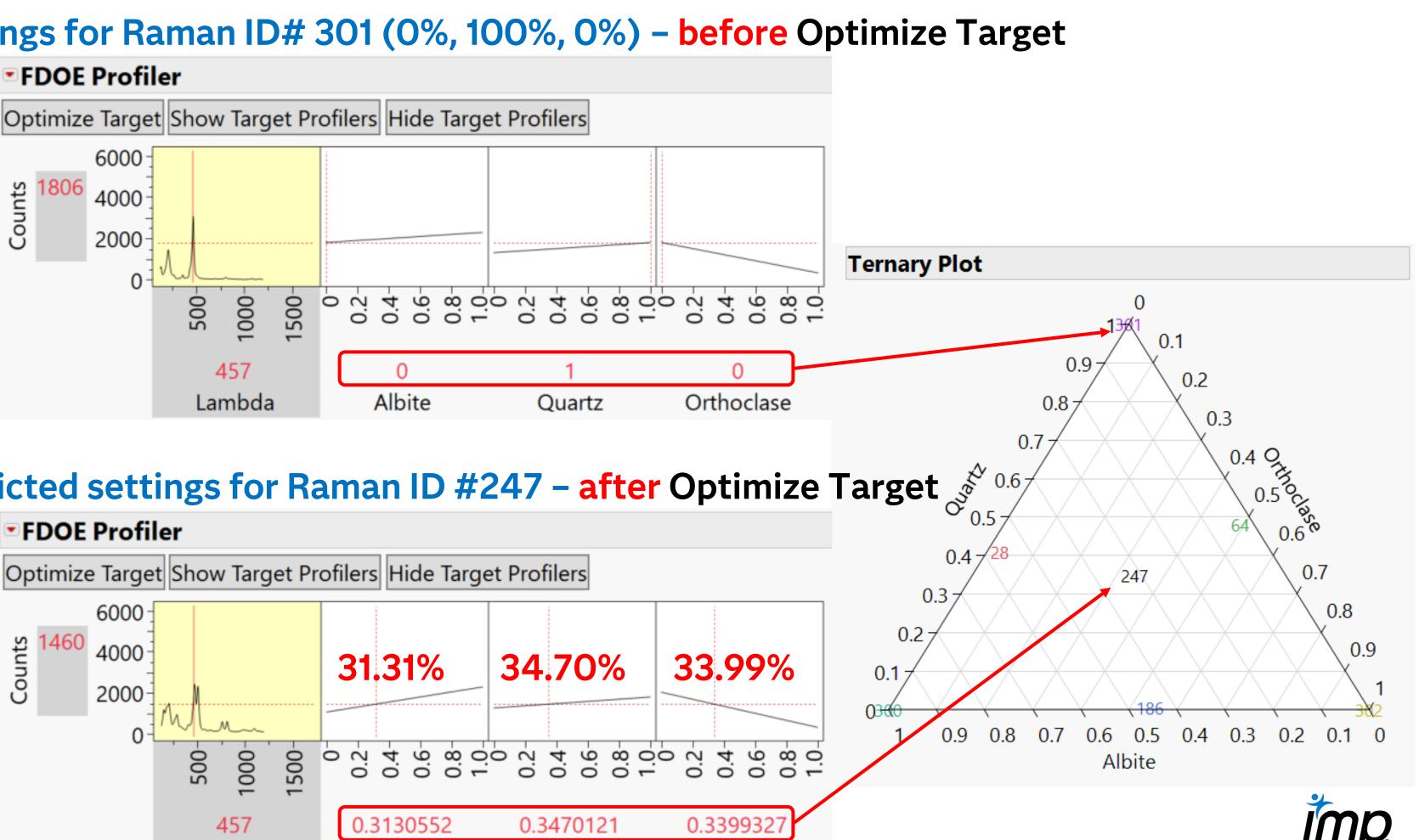


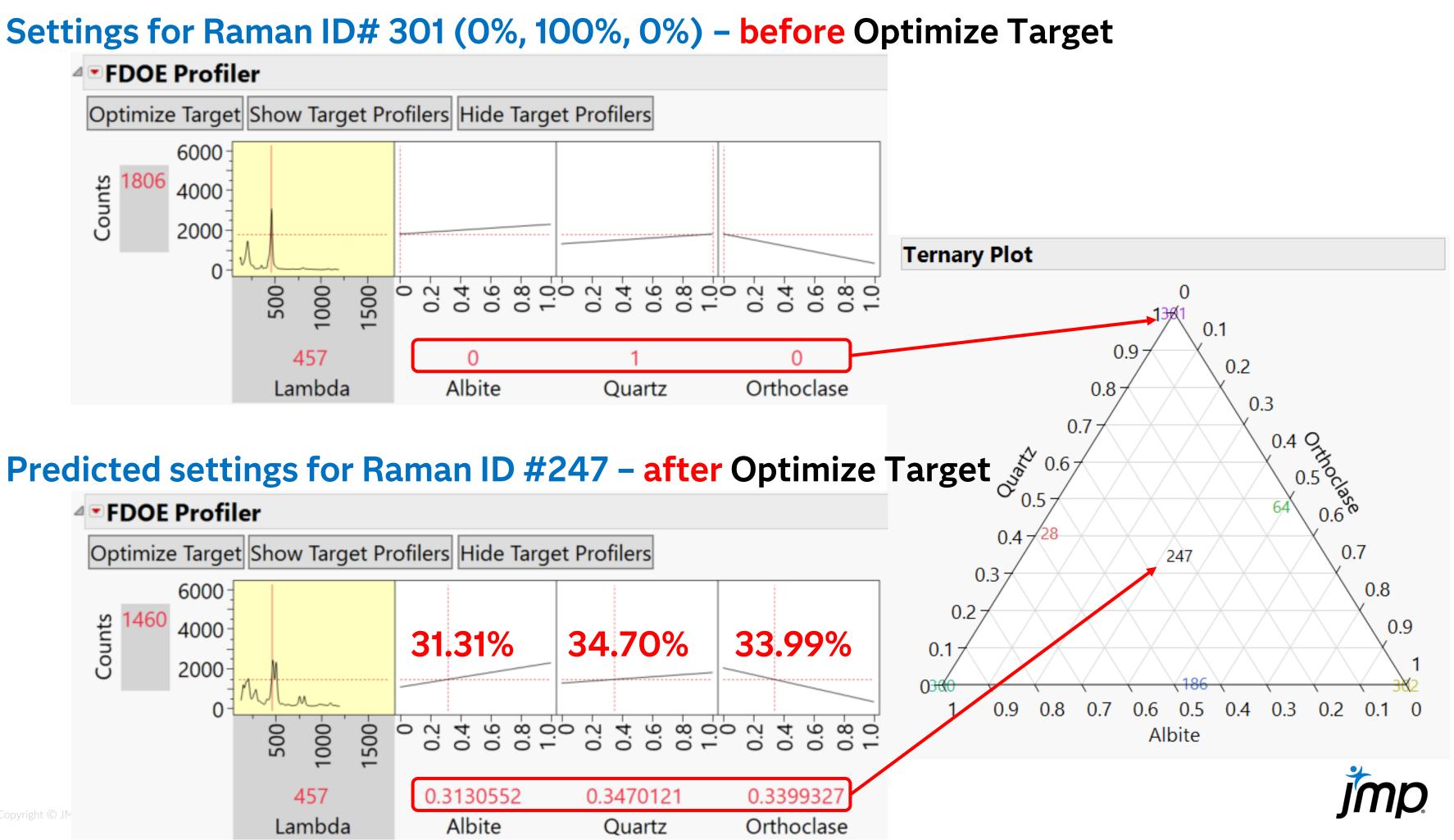
Case 2 - Reanalysis of Raman Spectral Data for 3-Mineral Mixture DOE using FDA

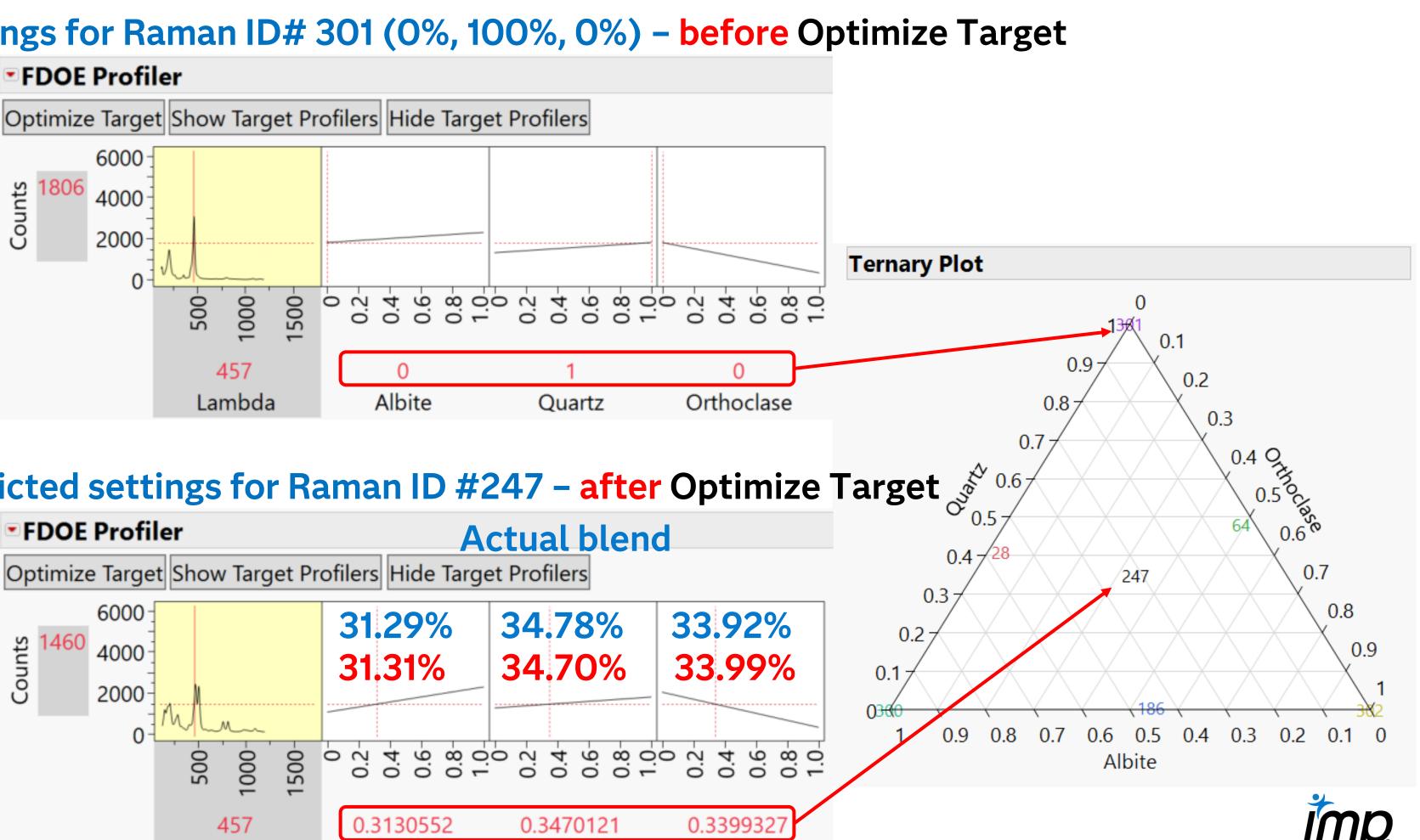
All 303 blends used as candidate trials 6 on right are resulting subset DOE

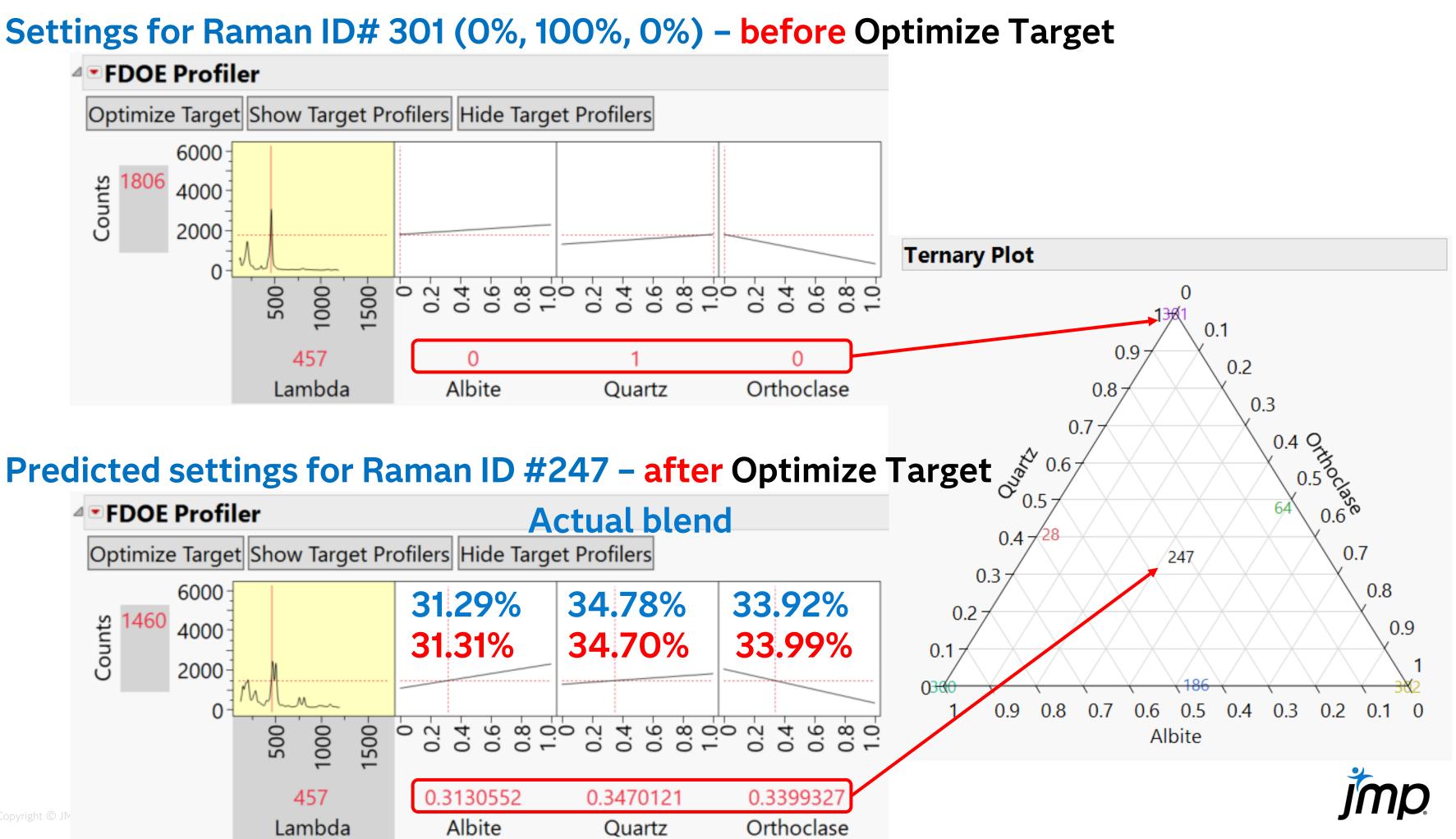






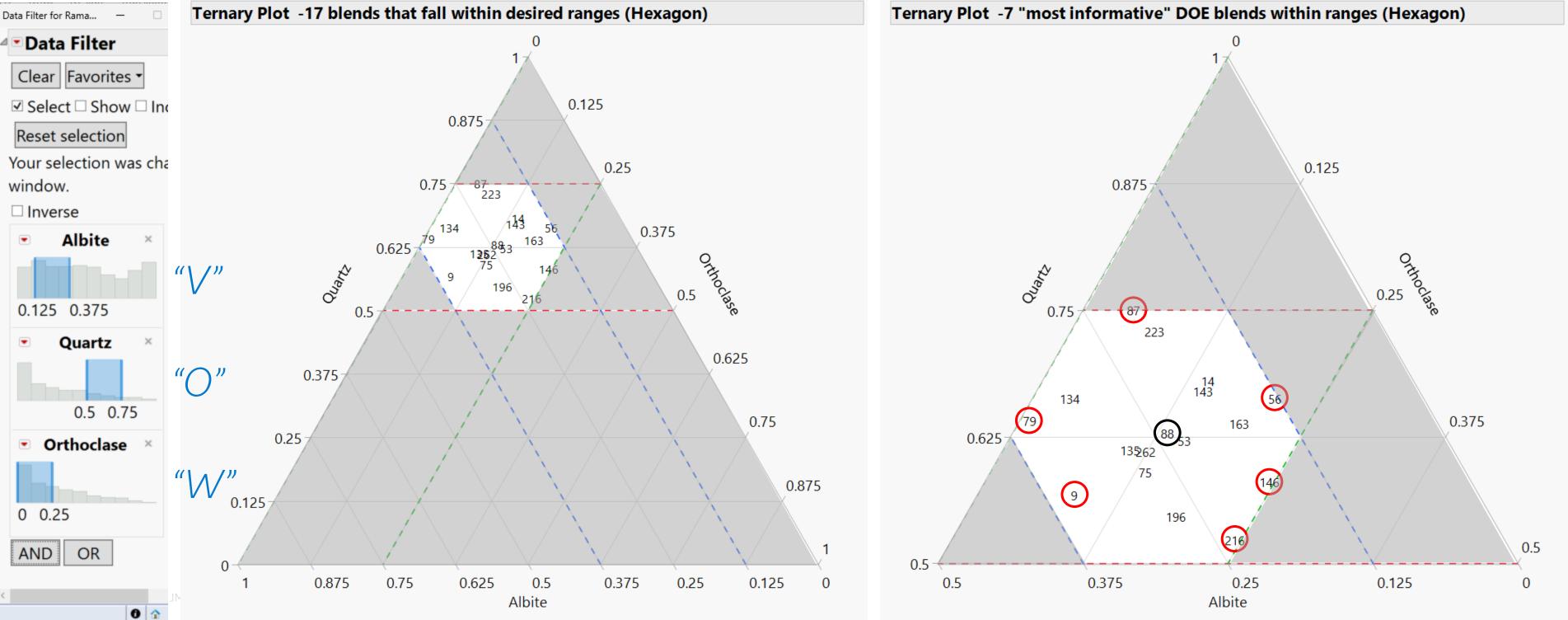






Case 2 - Reanalysis of Raman Spectral Data for 3-Mineral Mixture DOE using FDA

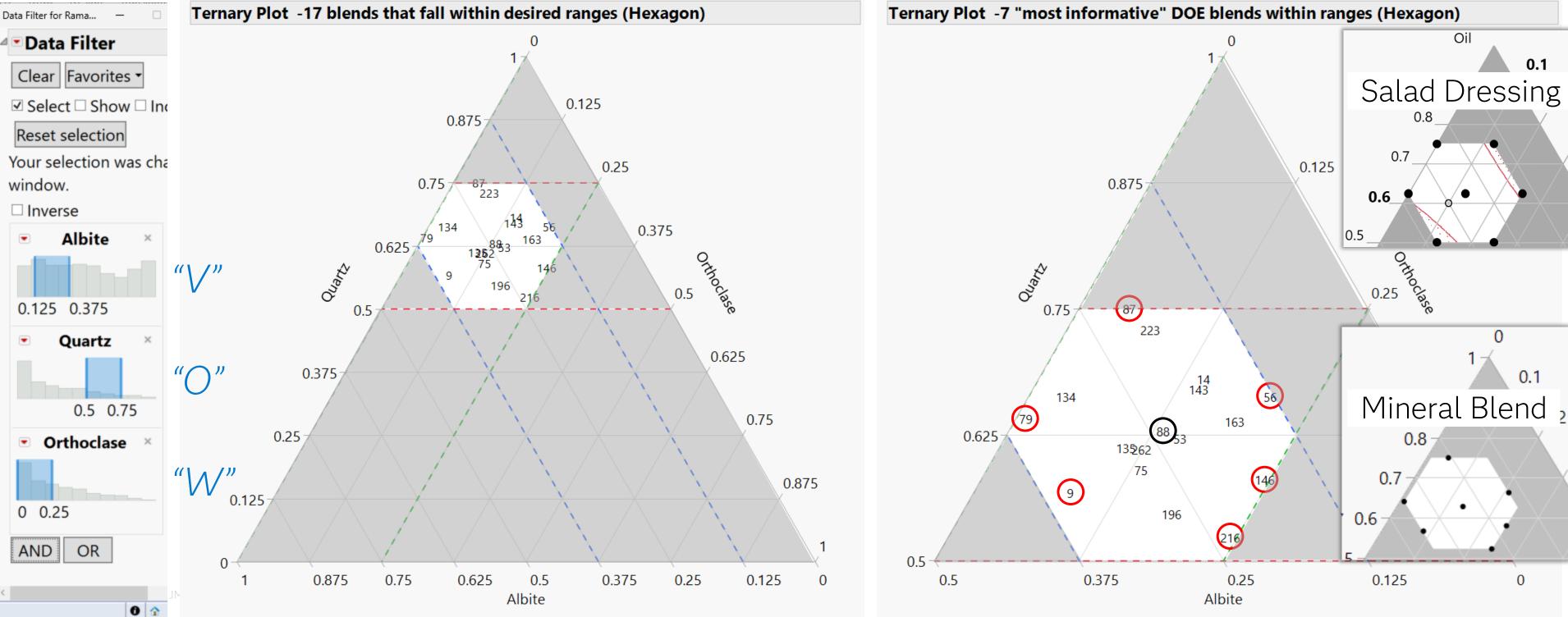
17 *filtered* blends used as candidate trials



7 circled trials are DOE subset

Case 2 - Reanalysis of Raman Spectral Data for 3-Mineral Mixture DOE using FDA

17 *filtered* blends used as candidate trials

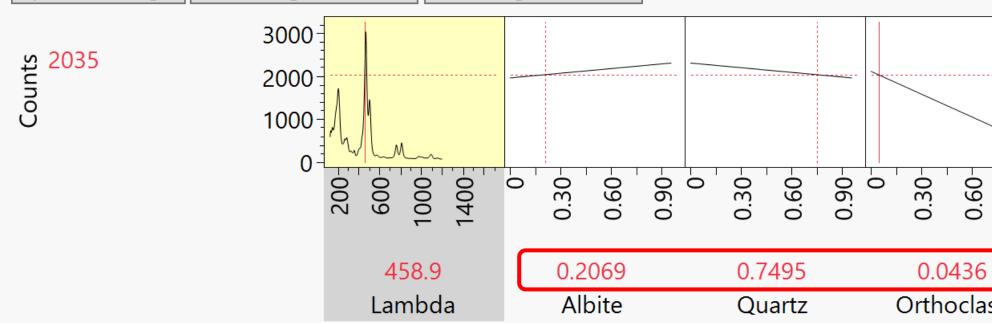


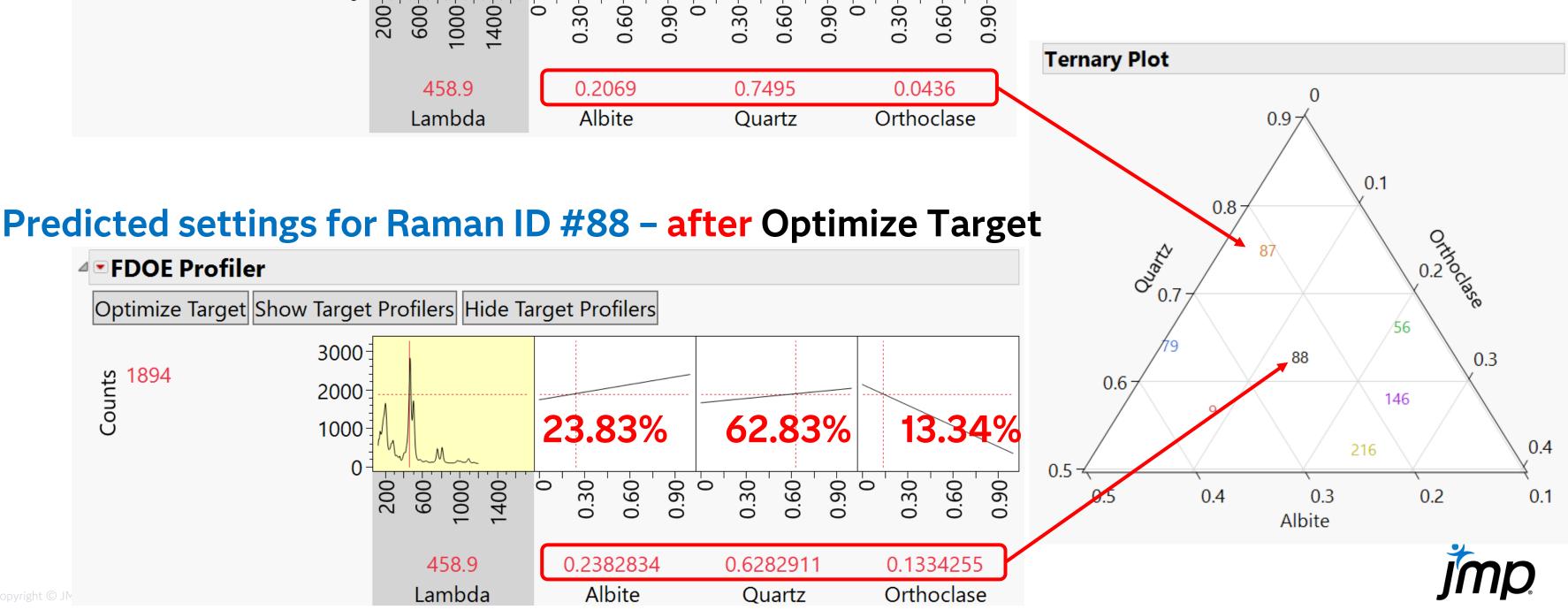
7 circled trials are DOE subset

Settings for Raman ID# 87 (20.69%, 74.95%, 4.36%) – before Optimize Target

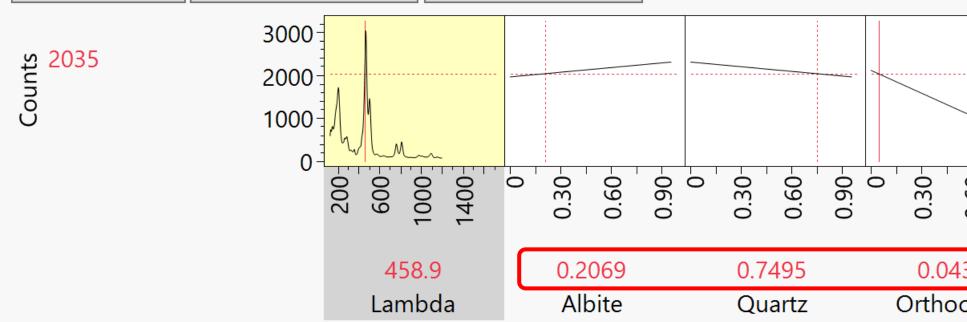
FDOE Profiler

Optimize Target Show Target Profilers Hide Target Profilers

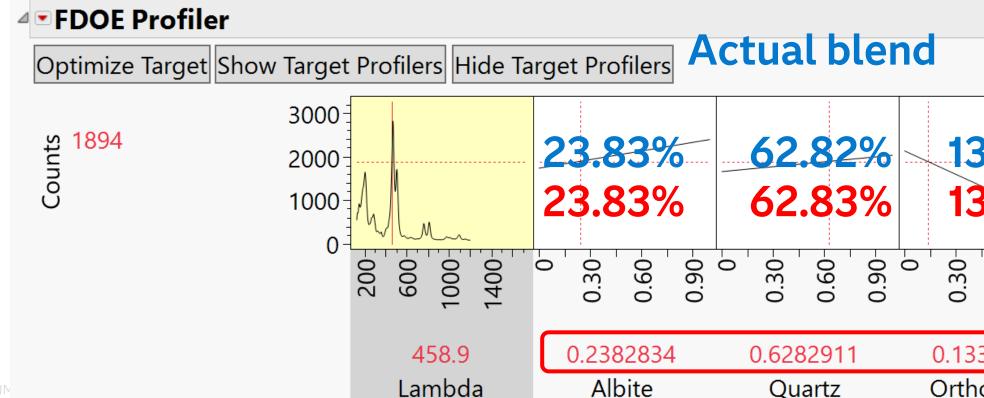




NOTE: Outer 6 points used to Train Model Raman ID# 88 used as Target spectra



Settings for Raman ID# 87 (20.69%, 74.95%, 4.36%) – before Optimize Target **FDOE** Profiler Optimize Target Show Target Profilers Hide Target Profilers 0.60 0.90 **Ternary Plot** 0.0436 Orthoclase 0.97 0.1 0.87 **Predicted settings for Raman ID #88 – after Optimize Target** Orthoclase Quarts 87 **✓ • FDOE Profiler Actual blend** 0.7 Optimize Target Show Target Profilers Hide Target Profilers /56 3000-88 0.3 Counts Counts 23.83% 62.82% 13.35% 0.67 2000-146 23.83% 62.83% 13.34% 1000 0.4 216 0 0.5 1400-0.60-1000 0.90-200--009 0.60 06.0 0.30-0.30 0.60 0.90 0.30 0 0 0 0.4 0.3 0.2 0.1 Albite 458.9 0.2382834 0.6282911 0.1334255 Orthoclase



Let's go to JMP...

- Filter out candidates in an interesting subregion of the full mixture space
- Use Custom DOE to find the most informative subset of 7 trials

of the full mixture space subset of 7 trials



Takeaways

- Use DOE to get most information from fewest trials 1.
- Use FDA to better model curves and spectra 2.
- Combine DOE with FDA to predict spectra 3.
- Combine DOE with FDA to predict formulation (factor settings) 4.
- Create a DOE by choosing informative subset of trials from candidates 5.



Questions?

Tom Donnelly

Principal Systems Engineer JMP Defense & Aerospace Team

tom.donnelly@jmp.com

Developer Tutorial Video on Spectral Data

Thanks to my JMP colleagues

Chris Gotwalt, Chief Data Scientist Ryan Parker, Sr Research Statistician Developer



STATISTICAL DISCOVERY

Copyright © JMP Statistical Discovery LLC. All rights reserved.