

JMP Statistical Discovery LLC

Assessing the Numerical Accuracy of JMP® 18



Measuring Accuracy

There are many sources of error in statistical computation, including rounding error, truncation error, and the finite-precision inaccuracy involved in representing a number in binary form.

To measure these sources of error, we looked at the significant digits reported by JMP calculations. Significant digits are the first nonzero digit and all succeeding digits. For example, 3.14159 has six significant digits, whereas 0.00314 has three. A frequently used measure of the number of correct significant digits is the (common) logarithm of the relative error (LRE). The LRE is calculated as follows:

$$\text{LRE} = -\log(|q - c| / |c|)$$

where q is the reported value and c is the expected value. This quantity is not defined when $q = c$. In that situation, the LRE is given the value of the number of significant digits in c . Also, there are situations where the expected value is zero, which also results in an undefined LRE. In these cases, the LRE is defined as the logarithm of the absolute error

$$\text{LRE} = -\log(|q|)$$

The LRE is approximately analogous to the number of significant digits of accuracy of a reported value compared with its expected value.

It is worth noting that the LRE is valid only for values of q that are close to c . Therefore, any calculated value that differs from c by more than a factor of 2 is set to zero. Finally, any value of the LRE greater than the number of digits in c is set to the number of digits in c .

We use the Greek symbol lambda (λ), with a subscript, to represent the LRE. The subscript denotes the parameter to be estimated. For example, λ_{rsq} is the LRE between a reported r square value and its expected value.

Statistical Standards

The National Institute of Standards and Technology (NIST) provides the Statistical Reference Data Sets (StRD) to assist in the evaluation of the numerical accuracy of statistical software. More information about these data sets is available at www.itl.nist.gov/div898/strd/. The StRD data sets are the subject of this paper.

The following sections report the results of tests that were run in JMP® 18 and JMP® Pro 18. All tests used the same build date: February 9, 2024. The tests were run for 64-bit systems on the following Windows versions: Windows 10 and Windows 11. The tests were run for 64-bit systems on the following macOS versions: Big Sur, Monterey, Ventura, and Sonoma.

Univariate Results

The univariate tests consist of nine data sets, ranging from 3 to 5000 data points. Each data set has certified values to 15 decimal places for the mean (μ), standard deviation (σ), and first-order autocorrelation (ρ). Therefore, a λ of 15 indicates perfect agreement with NIST certified values. Results for μ and σ were calculated using the JMP Distribution platform. Values of ρ came from the Time Series platform. The results are presented in Table 1. These results are the same for all Windows and macOS operating systems that we support.

Table 1: Univariate Results

| Data Set | Difficulty | λ_{μ} | λ_{σ} | λ_{ρ} |
|-----------|------------|-----------------|--------------------|------------------|
| PiDigits | Low | 15.0 | 14.9 | 13.0 |
| Lottery | Low | 15.0 | 15.0 | 15.0 |
| Lew | Low | 15.0 | 15.0 | 15.0 |
| Mavro | Low | 15.0 | 13.1 | 13.7 |
| Michaelso | Average | 15.0 | 13.8 | 13.4 |
| NumAcc1 | Average | 15.0 | 15.0 | 15.0 |
| NumAcc2 | Average | 14.0 | 14.6 | 13.6 |
| NumAcc3 | Average | 15.0 | 9.4 | 11.2 |
| NumAcc4 | High | 15.0 | 8.2 | 9.0 |

Analysis of Variance Results

The Analysis of Variance (ANOVA) tests contain eleven data sets, with 5 to 2001 values for each level. As in the case of the previous white paper, only the LREs of F , R^2 , and residual standard deviation are presented here. Again, 15 decimal places are provided in the certified values, so a score of 15 in the table corresponds to perfect agreement.

JMP provides two methods of calculating an ANOVA for two-variable cases. Its most direct method is through the Fit Y by X platform, designed specifically for bivariate data. The Fit Model platform, used for fitting general linear models, can also be used. Much of the literature on numerical accuracy reports results only for one method per software application. However, we report methods for both platforms because JMP uses distinct calculation routines for the two platforms. Results using the Fit Y by X platform are reported in Table 2, and results using the Fit Model platform are reported in Table 3. These results are the same for all Windows and macOS operating systems that we support.

Table 2: Analysis of Variance (ANOVA) Results Using Fit Y by X

| Data Set | Difficulty | λ_F | λ_{rstd} | λ_{rsq} |
|----------|------------|-------------|------------------|-----------------|
| SiRstv | Low | 12.3 | 13.1 | 12.4 |
| SmLs01 | Low | 13.9 | 14.4 | 14.2 |
| SmLs02 | Low | 13.4 | 13.8 | 13.7 |
| SmLs03 | Low | 12.3 | 12.8 | 12.6 |
| AtmAgWt | Average | 8.3 | 9.2 | 8.4 |
| SmLs04 | Average | 8.2 | 8.8 | 8.5 |
| SmLs05 | Average | 8.0 | 8.6 | 8.2 |
| SmLs06 | Average | 6.1 | 6.8 | 6.4 |
| SmLs07 | High | 2.4 | 3.0 | 2.7 |
| SmLs08 | High | 1.9 | 2.5 | 2.1 |
| SmLs09 | High | 0.3 | 0.8 | 0.5 |

Table 3: Analysis of Variance (ANOVA) Results Using Fit Model

| Data Set | Difficulty | λ_F | λ_{rstd} | λ_{rsq} |
|----------|------------|-------------|------------------|-----------------|
| SiRstv | Low | 13.0 | 13.4 | 13.1 |
| SmLs01 | Low | 14.4 | 15.0 | 14.6 |
| SmLs02 | Low | 13.7 | 14.1 | 14.0 |
| SmLs03 | Low | 12.4 | 12.9 | 12.6 |
| AtmAgWt | Average | 10.1 | 11.2 | 10.2 |
| SmLs04 | Average | 10.4 | 10.5 | 10.7 |
| SmLs05 | Average | 10.2 | 10.5 | 10.4 |
| SmLs06 | Average | 10.1 | 10.5 | 10.4 |
| SmLs07 | High | 4.3 | 4.5 | 4.6 |
| SmLs08 | High | 3.8 | 4.5 | 4.1 |
| SmLs09 | High | 3.4 | 4.5 | 3.7 |

Linear Regression Results

The linear regression portion of the test consists of eleven data sets containing 3 to 82 data points and 1 to 11 parameters to be estimated. Each data set contains values that have been certified to 15 digits. Certified values are available for each parameter's estimate, residual standard deviation R^2 , and the entire analysis of variance table (which provides the residual sum of squares).

As is the case with the ANOVA results, JMP provides two methods of fitting linear regressions: The Fit Y by X platform and the Fit Model platform. Results for each are provided for LREs of the parameter estimate β , its standard deviation s , and the residual sum of squares rss .

JMP also does not report an R^2 when the intercept is missing. This is the case in both the Fit Y by X and Fit Model platforms. Therefore, the two NoInt data sets have "NR" as their LRE for R^2 . Similarly, the Longley data set requires a multilinear fit, which is not available in the Fit Y by X platform. Therefore, results for Longley are listed as "NR" in the Fit Y by X tables (Table 4), but with LREs in the Fit Model tables (Table 5). Also, the Filip data requires a tenth degree polynomial fit that is not available in either the Fit Y by X or Fit Model platforms. Therefore, the results for Filip are listed as "NR" in all the following tables. These results are the same for all Windows and macOS operating systems that we support.

Table 4: Linear Regression Results Using Fit Y by X

| Data Set | Difficulty | Parameter | λ_{β} | λ_s | λ_{rss} | λ_{rsq} |
|----------|------------|-----------|-------------------|-------------|-----------------|-----------------|
| Norris | Low | β_0 | 12.0 | 10.8 | 10.8 | 15.0 |
| | | β_1 | 14.3 | 10.8 | | |
| Pontius | Low | β_0 | 11.2 | 8.4 | 8.4 | 15.0 |
| | | β_1 | 13.8 | 8.4 | | |
| | | β_2 | 12.1 | 8.4 | | |
| NoInt1 | Average | β_1 | 14.7 | 13.4 | 13.5 | NR |
| NoInt2 | Average | β_1 | 15.0 | 14.6 | 14.7 | NR |
| Filip | High | NR | NR | NR | NR | NR |
| Longley | High | NR | NR | NR | NR | NR |
| Wampler1 | High | β_0 | 8.4 | 15.0 | 15.0 | 15.0 |
| | | β_1 | 8.0 | 15.0 | | |
| | | β_2 | 8.4 | 15.0 | | |
| | | β_3 | 9.2 | 15.0 | | |

Table 4: Linear Regression Results Using Fit Y by X (Continued)

| Data Set | Difficulty | Parameter | λ_{β} | λ_s | λ_{rss} | λ_{rsq} |
|----------|------------|-----------|-------------------|-------------|-----------------|-----------------|
| | | β_4 | 10.4 | 15.0 | | |
| | | β_5 | 12.1 | 15.0 | | |
| Wampler2 | High | β_0 | 12.8 | 15.0 | 15.0 | 15.0 |
| | | β_1 | 11.8 | 15.0 | | |
| | | β_2 | 10.8 | 15.0 | | |
| | | β_3 | 10.5 | 15.0 | | |
| | | β_4 | 10.7 | 15.0 | | |
| | | β_5 | 11.3 | 15.0 | | |
| Wampler3 | High | β_0 | 8.4 | 11.3 | 11.7 | 15.0 |
| | | β_1 | 8.0 | 10.9 | | |
| | | β_2 | 8.4 | 10.8 | | |
| | | β_3 | 9.2 | 10.8 | | |
| | | β_4 | 10.4 | 10.8 | | |
| | | β_5 | 12.1 | 10.8 | | |
| Wampler4 | High | β_0 | 8.4 | 11.5 | 14.7 | 15.0 |
| | | β_1 | 8.0 | 10.9 | | |
| | | β_2 | 8.4 | 10.9 | | |
| | | β_3 | 9.2 | 10.8 | | |
| | | β_4 | 10.4 | 10.8 | | |
| | | β_5 | 12.1 | 10.8 | | |
| Wampler5 | High | β_0 | 8.4 | 11.5 | 14.8 | 13.7 |
| | | β_1 | 8.0 | 10.9 | | |
| | | β_2 | 8.4 | 10.9 | | |
| | | β_3 | 9.2 | 10.8 | | |
| | | β_4 | 10.4 | 10.8 | | |
| | | β_5 | 12.1 | 10.8 | | |

Table 5: Linear Regression Results Using Fit Model

| Data Set | Difficulty | Parameter | λ_{β} | λ_s | λ_{rstd} | λ_{rsq} |
|----------|------------|-----------|-------------------|-------------|------------------|-----------------|
| Norris | Low | β_0 | 12.4 | 10.8 | 10.8 | 15.0 |
| | | β_1 | 14.3 | 10.8 | | |
| Pontius | Low | β_0 | 11.5 | 9.3 | 9.3 | 15.0 |
| | | β_1 | 14.1 | 9.3 | | |
| | | β_2 | 12.5 | 9.3 | | |
| NoInt1 | Average | β_1 | 14.7 | 13.4 | 13.5 | NR |
| NoInt2 | Average | β_1 | 15.0 | 14.6 | 14.7 | NR |
| Filip | High | NR | NR | NR | NR | NR |
| Longley | High | β_0 | 13.5 | 14.7 | 13.5 | 15.0 |
| | | β_1 | 12.5 | 13.9 | | |
| | | β_2 | 12.9 | 13.6 | | |
| | | β_3 | 13.5 | 13.7 | | |
| | | β_4 | 14.0 | 13.6 | | |
| | | β_5 | 12.2 | 13.5 | | |
| | | β_6 | 13.5 | 14.6 | | |
| Wampler1 | High | β_0 | 6.7 | 15.0 | 15.0 | 15.0 |
| | | β_1 | 6.3 | 15.0 | | |
| | | β_2 | 6.8 | 15.0 | | |
| | | β_3 | 7.6 | 15.0 | | |
| | | β_4 | 8.9 | 15.0 | | |
| | | β_5 | 10.6 | 15.0 | | |
| Wampler2 | High | β_0 | 12.1 | 15.0 | 15.0 | 15.0 |
| | | β_1 | 11.0 | 15.0 | | |
| | | β_2 | 10.5 | 15.0 | | |
| | | β_3 | 10.5 | 15.0 | | |
| | | β_4 | 10.8 | 15.0 | | |
| | | β_5 | 11.6 | 15.0 | | |
| Wampler3 | High | β_0 | 6.7 | 10.7 | 10.8 | 15.0 |

Table 5: Linear Regression Results Using Fit Model (Continued)

| Data Set | Difficulty | Parameter | λ_{β} | λ_s | λ_{rstd} | λ_{rsq} |
|----------|------------|-----------|-------------------|-------------|-------------------------|------------------------|
| | | β_1 | 6.3 | 10.5 | | |
| | | β_2 | 6.8 | 10.4 | | |
| | | β_3 | 7.6 | 10.4 | | |
| | | β_4 | 8.9 | 10.4 | | |
| | | β_5 | 10.6 | 10.4 | | |
| Wampler4 | High | β_0 | 6.7 | 11.3 | 14.5 | 15.0 |
| | | β_1 | 6.3 | 10.7 | | |
| | | β_2 | 6.8 | 10.6 | | |
| | | β_3 | 7.6 | 10.6 | | |
| | | β_4 | 8.9 | 10.6 | | |
| | | β_5 | 10.6 | 10.6 | | |
| Wampler5 | High | β_0 | 6.7 | 11.3 | 14.8 | 13.7 |
| | | β_1 | 6.4 | 10.7 | | |
| | | β_2 | 6.9 | 10.6 | | |
| | | β_3 | 7.7 | 10.6 | | |
| | | β_4 | 9.0 | 10.6 | | |
| | | β_5 | 10.7 | 10.6 | | |

Nonlinear Regression Results

The Nonlinear regimen consists of twenty-seven data sets, with six to 250 data points and two to nine parameters. The certified values are presented to only eleven decimal places in this suite of tests, so an LRE of 11 implies perfect agreement with the standard.

- See Table 6 for Windows
- See Table 7 for macOS

Table 6: Nonlinear Regression Results for Windows

| Data Set | Difficulty | Parameter | λ_{β} | $\lambda_{\beta\text{std}}$ | λ_{sse} | λ_{rstd} |
|----------|------------|-----------|-------------------|-----------------------------|------------------------|-------------------------|
| Misra1a | Low | β_1 | 9.6 | 9.3 | 10.4 | 10.6 |
| | | β_2 | 9.6 | 10.6 | | |
| Chwirut2 | Low | β_1 | 7.4 | 7.9 | 11.0 | 10.8 |

Table 6: Nonlinear Regression Results for Windows (Continued)

| Data Set | Difficulty | Parameter | λ_{β} | $\lambda_{\beta\text{std}}$ | λ_{sse} | λ_{rstd} |
|----------|------------|-----------|-------------------|-----------------------------|------------------------|-------------------------|
| | | β_2 | 7.8 | 8.3 | | |
| | | β_3 | 7.7 | 9.0 | | |
| Chwirut1 | Low | β_1 | 9.3 | 9.7 | 11.0 | 10.9 |
| | | β_2 | 9.8 | 10.3 | | |
| | | β_3 | 9.6 | 11.0 | | |
| Lanczos3 | Low | β_1 | 9.2 | 8.3 | 10.5 | 11.0 |
| | | β_2 | 9.5 | 8.3 | | |
| | | β_3 | 9.9 | 8.3 | | |
| | | β_4 | 10.0 | 8.3 | | |
| | | β_5 | 10.2 | 8.3 | | |
| | | β_6 | 10.7 | 8.3 | | |
| Gauss1 | Low | β_1 | 11.0 | 11.0 | 11.0 | 11.0 |
| | | β_2 | 10.9 | 10.7 | | |
| | | β_3 | 11.0 | 11.0 | | |
| | | β_4 | 11.0 | 11.0 | | |
| | | β_5 | 10.6 | 11.0 | | |
| | | β_6 | 11.0 | 11.0 | | |
| | | β_7 | 10.8 | 11.0 | | |
| | | β_8 | 10.7 | 11.0 | | |
| Gauss2 | Low | β_1 | 11.0 | 10.7 | 10.6 | 10.6 |
| | | β_2 | 11.0 | 10.8 | | |
| | | β_3 | 10.3 | 10.9 | | |
| | | β_4 | 10.3 | 10.1 | | |
| | | β_5 | 10.4 | 10.9 | | |
| | | β_6 | 10.7 | 10.5 | | |
| | | β_7 | 11.0 | 9.9 | | |
| | | β_8 | 10.0 | 9.9 | | |
| DanWood | Low | β_1 | 10.0 | 10.1 | 11.0 | 11.0 |
| | | β_2 | 10.2 | 10.8 | | |

Table 6: Nonlinear Regression Results for Windows (Continued)

| Data Set | Difficulty | Parameter | λ_{β} | $\lambda_{\beta\text{std}}$ | λ_{sse} | λ_{rstd} |
|----------|------------|-----------|-------------------|-----------------------------|------------------------|-------------------------|
| Misra1b | Low | β_1 | 10.8 | 10.8 | 11.0 | 11.0 |
| | | β_2 | 10.8 | 11.0 | | |
| Kirby2 | Average | β_1 | 8.5 | 9.3 | 11.0 | 10.7 |
| | | β_2 | 8.6 | 9.0 | | |
| | | β_3 | 8.8 | 8.9 | | |
| | | β_4 | 8.6 | 8.8 | | |
| | | β_5 | 9.1 | 9.0 | | |
| Hahn1 | Average | β_1 | 8.4 | 9.1 | 10.5 | 11.0 |
| | | β_2 | 8.5 | 9.0 | | |
| | | β_3 | 8.8 | 9.0 | | |
| | | β_4 | 8.2 | 9.2 | | |
| | | β_5 | 9.6 | 9.4 | | |
| | | β_6 | 8.9 | 9.0 | | |
| | | β_7 | 8.4 | 9.2 | | |
| Nelson | Average | β_1 | 10.8 | 10.6 | 10.9 | 11.0 |
| | | β_2 | 10.9 | 11.0 | | |
| | | β_3 | 11.0 | 10.9 | | |
| MGH17 | Average | β_1 | 9.3 | 10.3 | 11.0 | 11.0 |
| | | β_2 | 7.9 | 7.3 | | |
| | | β_3 | 7.8 | 7.3 | | |
| | | β_4 | 8.4 | 7.7 | | |
| | | β_5 | 8.3 | 8.0 | | |
| Lanczos1 | Average | β_1 | 11.0 | 3.0 | 2.7 | 3.0 |
| | | β_2 | 10.5 | 3.0 | | |
| | | β_3 | 11.0 | 3.0 | | |
| | | β_4 | 10.9 | 3.0 | | |
| | | β_5 | 10.6 | 3.0 | | |
| | | β_6 | 11.0 | 3.0 | | |
| Lanczos2 | Average | β_1 | 11.0 | 8.3 | 9.9 | 10.5 |

Table 6: Nonlinear Regression Results for Windows (Continued)

| Data Set | Difficulty | Parameter | λ_{β} | $\lambda_{\beta\text{std}}$ | λ_{sse} | λ_{rstd} |
|----------|------------|-----------|-------------------|-----------------------------|------------------------|-------------------------|
| | | β_2 | 10.3 | 8.4 | | |
| | | β_3 | 11.0 | 8.3 | | |
| | | β_4 | 11.0 | 8.3 | | |
| | | β_5 | 10.7 | 8.3 | | |
| | | β_6 | 11.0 | 8.3 | | |
| Gauss3 | Average | β_1 | 11.0 | 11.0 | 10.9 | 10.7 |
| | | β_2 | 10.9 | 10.4 | | |
| | | β_3 | 10.6 | 10.4 | | |
| | | β_4 | 10.5 | 11.0 | | |
| | | β_5 | 11.0 | 11.0 | | |
| | | β_6 | 11.0 | 10.6 | | |
| | | β_7 | 10.4 | 10.9 | | |
| | | β_8 | 10.6 | 11.0 | | |
| Misra1c | Average | β_1 | 11.0 | 11.0 | 11.0 | 11.0 |
| | | β_2 | 10.8 | 10.6 | | |
| Misra1d | Average | β_1 | 11.0 | 10.8 | 11.0 | 11.0 |
| | | β_2 | 11.0 | 11.0 | | |
| Roszman1 | Average | β_1 | 11.0 | 11.0 | 11.0 | 11.0 |
| | | β_2 | 10.4 | 10.9 | | |
| | | β_3 | 10.8 | 11.0 | | |
| | | β_4 | 10.9 | 10.7 | | |
| Enso | Average | β_1 | 9.9 | 9.7 | 11.0 | 11.0 |
| | | β_2 | 9.4 | 10.6 | | |
| | | β_3 | 8.7 | 10.3 | | |
| | | β_4 | 8.8 | 8.2 | | |
| | | β_5 | 8.2 | 8.0 | | |
| | | β_6 | 7.4 | 8.6 | | |
| | | β_7 | 8.5 | 8.0 | | |
| | | β_8 | 6.4 | 8.8 | | |

Table 6: Nonlinear Regression Results for Windows (Continued)

| Data Set | Difficulty | Parameter | λ_{β} | $\lambda_{\beta\text{std}}$ | λ_{sse} | λ_{rstd} |
|----------|------------|-----------|-------------------|-----------------------------|------------------------|-------------------------|
| | | β_9 | 8.1 | 7.6 | | |
| Mgh09 | High | β_1 | 8.7 | 8.0 | 11.0 | 11.0 |
| | | β_2 | 7.4 | 7.5 | | |
| | | β_3 | 7.9 | 7.6 | | |
| | | β_4 | 7.5 | 7.6 | | |
| Thurber | High | β_1 | 10.2 | 9.0 | 11.0 | 10.6 |
| | | β_2 | 8.4 | 7.0 | | |
| | | β_3 | 8.1 | 7.0 | | |
| | | β_4 | 8.0 | 7.0 | | |
| | | β_5 | 8.4 | 7.0 | | |
| | | β_6 | 8.3 | 7.0 | | |
| | | β_7 | 7.6 | 7.0 | | |
| BoxBOD | High | β_1 | 9.8 | 9.4 | 10.4 | 11.0 |
| | | β_2 | 9.2 | 9.2 | | |
| Rat42 | High | β_1 | 9.6 | 8.9 | 11.0 | 10.4 |
| | | β_2 | 9.8 | 9.3 | | |
| | | β_3 | 9.3 | 9.4 | | |
| Mgh10 | High | β_1 | 11.0 | 9.7 | 11.0 | 11.0 |
| | | β_2 | 11.0 | 9.8 | | |
| | | β_3 | 10.8 | 9.8 | | |
| Eckerle4 | High | β_1 | 10.0 | 9.8 | 10.7 | 11.0 |
| | | β_2 | 9.6 | 9.6 | | |
| | | β_3 | 10.9 | 9.6 | | |
| Rat43 | High | β_1 | 9.6 | 8.7 | 11.0 | 11.0 |
| | | β_2 | 8.4 | 8.6 | | |
| | | β_3 | 8.5 | 8.4 | | |
| | | β_4 | 8.4 | 8.6 | | |
| Bennett5 | High | β_1 | 7.3 | 6.6 | 11.0 | 10.6 |
| | | β_2 | 8.0 | 6.6 | | |

Table 6: Nonlinear Regression Results for Windows (Continued)

| Data Set | Difficulty | Parameter | λ_{β} | $\lambda_{\beta\text{std}}$ | λ_{sse} | λ_{rstd} |
|----------|------------|-----------|-------------------|-----------------------------|------------------------|-------------------------|
| | | β_3 | 8.0 | 6.6 | | |

Table 7: Nonlinear Regression Results for macOS

| Data Set | Difficulty | Parameter | λ_{β} | $\lambda_{\beta\text{std}}$ | λ_{sse} | λ_{rstd} |
|----------|------------|-----------|-------------------|-----------------------------|------------------------|-------------------------|
| Misra1a | Low | β_1 | 8.3 | 7.9 | 10.4 | 10.6 |
| | | β_2 | 8.2 | 9.4 | | |
| Chwirut2 | Low | β_1 | 7.7 | 8.1 | 11.0 | 10.8 |
| | | β_2 | 8.0 | 8.6 | | |
| | | β_3 | 8.0 | 9.3 | | |
| Chwirut1 | Low | β_1 | 10.5 | 10.7 | 11.0 | 10.9 |
| | | β_2 | 11.0 | 10.8 | | |
| | | β_3 | 11.0 | 11.0 | | |
| Lanczos3 | Low | β_1 | 8.3 | 8.5 | 10.5 | 11.0 |
| | | β_2 | 8.6 | 8.5 | | |
| | | β_3 | 8.9 | 8.3 | | |
| | | β_4 | 9.1 | 8.5 | | |
| | | β_5 | 9.1 | 8.4 | | |
| | | β_6 | 9.8 | 8.4 | | |
| Gauss1 | Low | β_1 | 11.0 | 9.6 | 11.0 | 11.0 |
| | | β_2 | 8.9 | 8.7 | | |
| | | β_3 | 11.0 | 9.8 | | |
| | | β_4 | 11.0 | 11.0 | | |
| | | β_5 | 10.6 | 9.8 | | |
| | | β_6 | 11.0 | 10.6 | | |
| | | β_7 | 10.8 | 10.9 | | |
| | | β_8 | 10.5 | 10.3 | | |
| Gauss2 | Low | β_1 | 11.0 | 10.7 | 10.6 | 10.6 |
| | | β_2 | 11.0 | 10.8 | | |

Table 7: Nonlinear Regression Results for macOS (Continued)

| Data Set | Difficulty | Parameter | λ_{β} | $\lambda_{\beta\text{std}}$ | λ_{sse} | λ_{rstd} |
|----------|------------|-----------|-------------------|-----------------------------|------------------------|-------------------------|
| | | β_3 | 10.3 | 10.9 | | |
| | | β_4 | 10.3 | 10.1 | | |
| | | β_5 | 10.4 | 10.9 | | |
| | | β_6 | 10.7 | 10.5 | | |
| | | β_7 | 11.0 | 9.9 | | |
| | | β_8 | 10.0 | 9.9 | | |
| DanWood | Low | β_1 | 10.0 | 10.1 | 11.0 | 11.0 |
| | | β_2 | 10.2 | 10.8 | | |
| Misra1b | Low | β_1 | 10.9 | 10.8 | 11.0 | 11.0 |
| | | β_2 | 10.9 | 11.0 | | |
| Kirby2 | Average | β_1 | 8.5 | 9.3 | 11.0 | 10.7 |
| | | β_2 | 8.6 | 9.0 | | |
| | | β_3 | 8.8 | 8.9 | | |
| | | β_4 | 8.6 | 8.8 | | |
| | | β_5 | 9.1 | 9.0 | | |
| Hahn1 | Average | β_1 | 8.4 | 9.1 | 10.5 | 11.0 |
| | | β_2 | 8.5 | 9.0 | | |
| | | β_3 | 8.8 | 9.0 | | |
| | | β_4 | 8.2 | 9.2 | | |
| | | β_5 | 9.6 | 9.4 | | |
| | | β_6 | 8.9 | 9.0 | | |
| | | β_7 | 8.4 | 9.2 | | |
| Nelson | Average | β_1 | 10.8 | 10.6 | 10.9 | 11.0 |
| | | β_2 | 10.9 | 11.0 | | |
| | | β_3 | 11.0 | 10.9 | | |
| MGH17 | Average | β_1 | 11.0 | 10.8 | 11.0 | 11.0 |
| | | β_2 | 9.9 | 9.4 | | |
| | | β_3 | 9.8 | 9.5 | | |
| | | β_4 | 10.4 | 9.9 | | |

Table 7: Nonlinear Regression Results for macOS (Continued)

| Data Set | Difficulty | Parameter | λ_{β} | $\lambda_{\beta\text{std}}$ | λ_{sse} | λ_{rstd} |
|----------|------------|-----------|-------------------|-----------------------------|------------------------|-------------------------|
| | | β_5 | 10.3 | 10.5 | | |
| Lanczos1 | Average | β_1 | 11.0 | 3.1 | 2.7 | 3.1 |
| | | β_2 | 10.5 | 3.1 | | |
| | | β_3 | 11.0 | 3.1 | | |
| | | β_4 | 10.9 | 3.1 | | |
| | | β_5 | 10.6 | 3.1 | | |
| | | β_6 | 11.0 | 3.1 | | |
| Lanczos2 | Average | β_1 | 11.0 | 8.3 | 10.0 | 10.6 |
| | | β_2 | 10.3 | 8.3 | | |
| | | β_3 | 11.0 | 8.3 | | |
| | | β_4 | 11.0 | 8.3 | | |
| | | β_5 | 10.7 | 8.3 | | |
| | | β_6 | 11.0 | 8.3 | | |
| Gauss3 | Average | β_1 | 11.0 | 11.0 | 10.9 | 10.7 |
| | | β_2 | 10.9 | 10.4 | | |
| | | β_3 | 10.6 | 10.4 | | |
| | | β_4 | 10.5 | 10.6 | | |
| | | β_5 | 11.0 | 10.9 | | |
| | | β_6 | 11.0 | 10.6 | | |
| | | β_7 | 10.4 | 10.6 | | |
| | | β_8 | 10.6 | 10.9 | | |
| Misra1c | Average | β_1 | 11.0 | 11.0 | 11.0 | 11.0 |
| | | β_2 | 10.8 | 10.6 | | |
| Misra1d | Average | β_1 | 11.0 | 10.8 | 11.0 | 11.0 |
| | | β_2 | 11.0 | 11.0 | | |
| Roszman1 | Average | β_1 | 8.3 | 9.0 | 11.0 | 11.0 |
| | | β_2 | 7.6 | 8.9 | | |
| | | β_3 | 8.5 | 8.9 | | |
| | | β_4 | 7.9 | 8.1 | | |

Table 7: Nonlinear Regression Results for macOS (Continued)

| Data Set | Difficulty | Parameter | λ_{β} | $\lambda_{\beta\text{std}}$ | λ_{sse} | λ_{rstd} |
|----------|------------|-----------|-------------------|-----------------------------|------------------------|-------------------------|
| Enso | Average | β_1 | 10.2 | 10.0 | 11.0 | 11.0 |
| | | β_2 | 9.8 | 11.0 | | |
| | | β_3 | 9.0 | 10.6 | | |
| | | β_4 | 9.1 | 8.5 | | |
| | | β_5 | 8.6 | 8.3 | | |
| | | β_6 | 7.7 | 8.9 | | |
| | | β_7 | 8.8 | 8.3 | | |
| | | β_8 | 6.8 | 9.1 | | |
| | | β_9 | 8.5 | 7.9 | | |
| Mgh09 | High | β_1 | 8.7 | 8.0 | 11.0 | 11.0 |
| | | β_2 | 7.4 | 7.5 | | |
| | | β_3 | 7.9 | 7.6 | | |
| | | β_4 | 7.5 | 7.6 | | |
| Thurber | High | β_1 | 10.2 | 8.9 | 11.0 | 10.6 |
| | | β_2 | 8.3 | 6.9 | | |
| | | β_3 | 8.0 | 6.9 | | |
| | | β_4 | 7.9 | 6.9 | | |
| | | β_5 | 8.3 | 6.9 | | |
| | | β_6 | 8.2 | 6.9 | | |
| | | β_7 | 7.5 | 6.9 | | |
| BoxBOD | High | β_1 | 9.8 | 9.4 | 10.4 | 11.0 |
| | | β_2 | 9.2 | 9.2 | | |
| Rat42 | High | β_1 | 9.6 | 8.9 | 11.0 | 10.4 |
| | | β_2 | 9.8 | 9.3 | | |
| | | β_3 | 9.3 | 9.4 | | |
| Mgh10 | High | β_1 | 11.0 | 9.7 | 11.0 | 11.0 |
| | | β_2 | 11.0 | 9.8 | | |
| | | β_3 | 10.8 | 9.8 | | |
| Eckerle4 | High | β_1 | 10.0 | 9.8 | 10.7 | 11.0 |

Table 7: Nonlinear Regression Results for macOS (Continued)

| Data Set | Difficulty | Parameter | λ_{β} | $\lambda_{\beta\text{std}}$ | λ_{sse} | λ_{rstd} |
|----------|------------|-----------|-------------------|-----------------------------|------------------------|-------------------------|
| | | β_2 | 9.6 | 9.6 | | |
| | | β_3 | 10.9 | 9.6 | | |
| Rat43 | High | β_1 | 10.2 | 9.3 | 11.0 | 11.0 |
| | | β_2 | 9.0 | 9.2 | | |
| | | β_3 | 9.2 | 9.0 | | |
| | | β_4 | 9.0 | 9.2 | | |
| Bennett5 | High | β_1 | 7.3 | 6.3 | 11.0 | 10.6 |
| | | β_2 | 8.0 | 6.3 | | |
| | | β_3 | 8.0 | 6.3 | | |

Appendix 1 Replicating JMP 18 Numerical Accuracy Results

To reproduce the results reported in these tables:

1. Download the compressed archive of the NIST testing framework (available at www.jmp.com/qualitystatement/).
2. Uncompress the archive to a directory of your choice.
3. Locate and execute the `RunNISTTests.jsl` script. The script creates the window shown in Figure 1. This window enables you to execute either all tests or selected tests. See Figure 2 for an example of the report that is generated when the tests are run.

The NIST testing framework can facilitate operational qualification or validation of JMP. The testing framework displays the LRE and the certified minimum LRE for each reported value. The testing framework also displays the relative error threshold (RET) value; the RET value is calculated based on the certified minimum LRE. The RET represents the minimum computational accuracy that we deem acceptable for our software.

If the LRE is greater than or equal to the certified minimum LRE, the test is passed. The resulting report shows the status of the test, the NIST standard value, the value actually computed, the LRE, the certified minimum LRE, and the RET. (The RET appears in a hidden column.) In this way, the tests serve as a tool to demonstrate that JMP is operating as expected.

Note: This framework is a specialized version of the framework described in the white paper titled “Unit Tests: Automated JSL Testing”. The framework consists of a set of JSL scripts and JMP data tables corresponding to each of the StRD data sets mentioned in the previous sections. See Appendix 2 for additional details.

Figure 1: NIST Test Driver

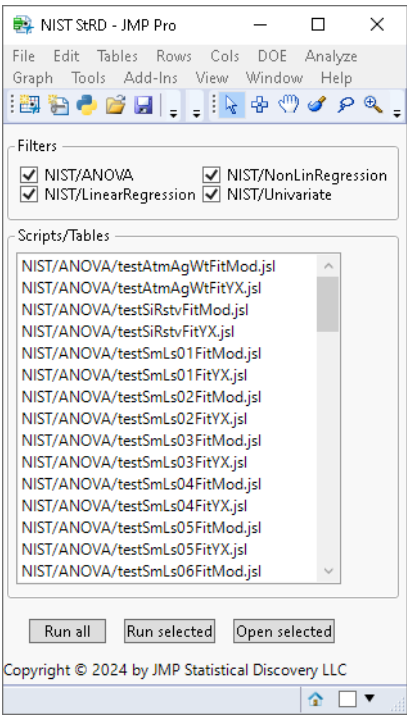
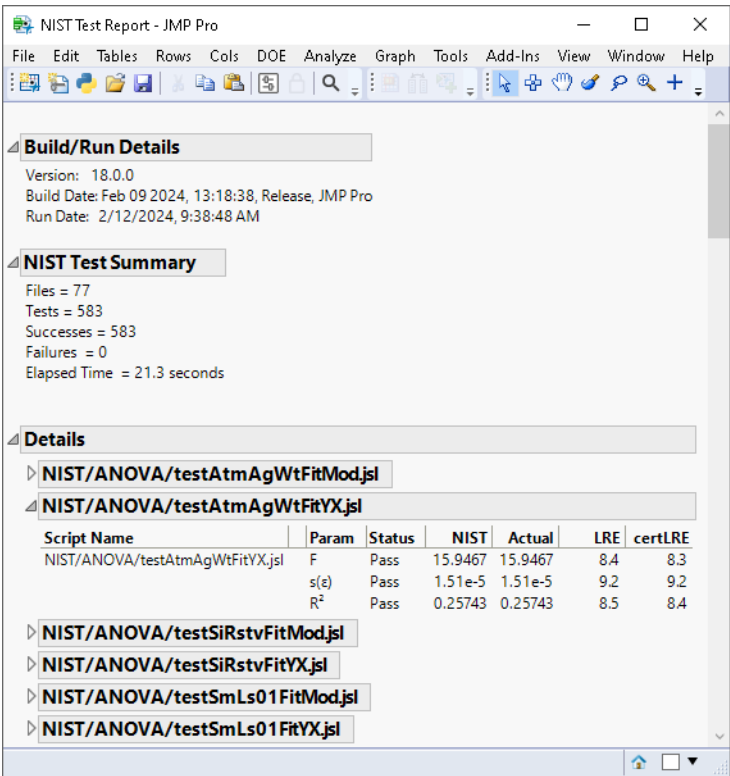


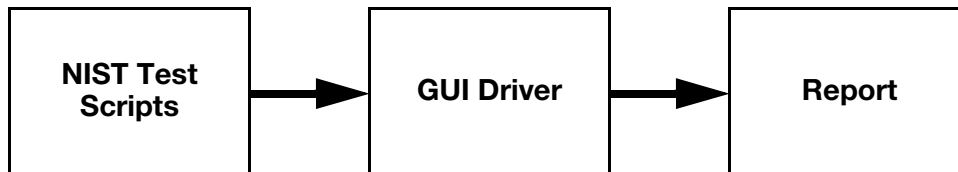
Figure 2: NIST Test Report



Appendix 2 NIST Testing Framework

The NIST framework consists of a set of JSL scripts and JMP data tables. It is intended to be host independent and should work for any version of JMP beginning with version 7. The architecture is as follows:

Figure 3: NIST Testing Framework



The GUI driver script (`RunNISTTests.jsl`) can reside anywhere in your file system. The NIST test scripts and associated data tables must reside in a subdirectory, named `tests/NIST/`, of the directory where the driver script is located. In addition, to be recognized by the driver, test scripts use a prefix of `test` (for example, `testMavro.jsl`).

NIST tests are specified as JSL scripts that access test data from JMP data tables. Individual test cases (for example, parameter estimates) are specified by way of a function named `ut assert LRE()` that is defined by the GUI driver. The prototype of the function is

```
ut assert LRE( expression, expected value, minimum LRE )
```

where the `expression` argument specifies the actual result and the `expected value` argument the expected result. A test case is considered a success (or a pass) if the computed LRE is greater than or equal to the value specified by the `minimum LRE` argument. Test cases also pass if the expected and actual values are both missing.

Note that the framework is a JSL application and the scripts that constitute the framework are provided in unencrypted form. Users can therefore change the driver script, associated utility scripts, or the test scripts, if necessary. Users can also add additional test scripts to the framework. As long as a few simple conventions (described in the following section) are followed, the driver automatically detects these scripts and makes them available for execution.

Adding Test Scripts to the Framework

1. Use the pattern below as a guide when writing your script.
2. Ensure that the script is stored in the `tests/NIST/` subdirectory.
3. Ensure that the script name has the `test` prefix (for example, `testMavro.jsl`).

NIST Test Script Pattern

```
Names Default to Here( 1 );
// Open data table
```

```
dt = Open( < name > );

// Expected results and certified LRE values
expected = < expected value >;
certLRE = < certified minimum LRE value >;

// Launch platform and define a reference to the report
obj = < platform launch expression >;
rpt = obj << report;

// Navigate display tree and get actual results
actual = rpt[ < subscript > ] << get as matrix;

// Invoke ut assert LRE function to execute numerical accuracy tests
ut assert LRE( expr(actual), expected, certLRE );

close(dt, no save);
wait(0);    // give window a chance to close
```

References

Creighton, L. and Ding, J. (2000). "Assessing the numerical accuracy of JMP". *SAS White Paper*.

McCulloch, B. D. (1998). "Assessing the reliability of statistical software: Part I". *The American Statistician*, v52, n4 (November 1998), 358-366.

Morgan, J. and Gregg, X. (2007). "Unit Tests: Automated JSL Testing". *JMP White Paper*. Available at <https://www.jmp.com/content/dam/jmp/documents/en/white-papers/unit-test-framework.pdf>.